Disagreement about Inflation and the Yield Curve by Paul Ehling, Michael Gallmeyer, Christian Heyerdahl-Larsen, Philipp Illeditsch

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Outline

- Why care about the term structure (real and nominal)
- Bond yields and the SDF
- Real yields and inflation
- Main theoretical result
- Comments

Why care about the term structure of interest rates?

Term structure tells us

- nominal rates at which the government can borrow and the rate at which investors are willing lend to the government
- crucial for investment decisions of firms, savings decisions of consumers, and policy decisions.

Can use yield spreads for forecasting

- future short yields (Campbell and Shiller, 1991; Cochrane and Piazzesi, 2005; Fama and Bliss, 1987)
- real activity (Ang et al., 2006; Estrella and Hardouvelis, 1991; Hamilton and Kim, 2002; Harvey, 1988) and inflation (Fama, 1990; Mishkin, 1990)

Monetary policy:

- central bank can move the short end of the yield curve, but long term yields important for aggregate demand, e.g. long term mortgage rates
- term structure model: short term yields $\rightarrow$ long term yields

Debt policy:

- how much to issue and when.
- how does this impact term structure?

This paper

- Inflation impacts real yields
- Why?
  - Differences in beliefs about inflation lead to consumption risk sharing across agents.
  - Consumption allocations impacted by differences in beliefs about inflation.
  - Real SDF impacted by differences in beliefs about inflation.

Bond yields and the SDF

- real and nominal SDF

$$\overbrace{\xi^*(t)}^{\text{nominal SDF}} = \frac{\overbrace{\xi(t)}^{\text{real SDF}}}{\underbrace{\pi(t)}_{\text{price level}}}$$

- nominal zero cpn bond

$$\begin{aligned} P(t,T) &= E_t\left[\frac{\xi(T)}{\xi(t)}\frac{\pi(t)}{\pi(T)}\right] \\ &= E_t\left[\frac{\xi(T)}{\xi(t)}\right] E_t\left[\frac{\pi(t)}{\pi(T)}\right] + Cov_t\left[\frac{\xi(T)}{\xi(t)}, \frac{\pi(t)}{\pi(T)}\right] \\ &= \underbrace{B(t,T)}_{\text{real zero cpn bond}} \; E_t\left[e^{-\overbrace{I(t,T)}^{\text{inflation}}}\right] + \underbrace{Cov_t\left[\frac{\xi(T)}{\xi(t)}, e^{-I(t,T)}\right]}_{\text{inflation risk premium}} \end{aligned}$$

Inflation and bond prices

$$P(t,T) = \underbrace{B(t,T)}_{\text{real zero cpn bond}} \; E_t\left[e^{-\overbrace{I(t,T)}^{\text{inflation}}}\right] + \underbrace{Cov_t\left[\frac{\xi(T)}{\xi(t)}, e^{-I(t,T)}\right]}_{\text{inflation risk premium}} \quad (1)$$

- Implications
  - High expected inflation → lower nominal bond price
  - future inflation high in bad states → lower nominal bond price

Real yields and inflation

- Empirical results in this paper: differences in beliefs about **inflation**
  - Greater differences in beliefs: higher nominal yield, higher vol
  - Greater differences in beliefs: higher **real** yield, higher vol
  - $B(t, T) = E_t \left[\frac{\xi(T)}{\xi(t)} \right]$ impacted by inflation expectations!

- Question: How can we justify this in a consumption - based equilibrium model?
  - How can differences in beliefs about inflation impact the value of a real cash flow when consumption is exogenous?
  - Obvious response: consumption and inflation are correlated.
  - This paper has a different answer

Model

- Output and price level

$$\frac{d\epsilon(t)}{\epsilon(t)} = \mu_\epsilon dt + \sigma_\epsilon dz_\epsilon(t),$$
$$d\pi(t) = \pi(t)x(t)dt + \sigma_{\pi,\epsilon} dz_\epsilon(t) + \sigma_{\pi,USD} dz_{USD}(t)$$

    - x is mean reverting: differences in beliefs about rate of mean reversion and long run mean
- Preferences

$$e^{-\rho t}\frac{1}{1-\gamma}\left(\frac{c_k(t)}{x(t)}\right)^{1-\gamma}$$

- Complete markets: FOC of rep agent

$$\lambda_{1,0} m_1(t) e^{-\rho t} c_1(t)^{-\gamma} x(t)^{-(\gamma-1)} = \lambda_{2,0} m_2(t) e^{-\rho t} c_2(t)^{-\gamma} x(t)^{-(\gamma-1)}$$

- $m_k(t) = \frac{d\mathbb{P}_i(s)}{d\mathbb{P}(s)}$, probability ratio (Radon - Nikodym derivative), exponential martingale
- γ plays double role
    - relative risk aversion: aversion to changes in consumption across states
    - reciprocal of EIS: aversion to changes in consumption over time

Theoretical result: SDF and Differences in beliefs

- FOC $\rightarrow$ consumption sharing rule

$$\begin{aligned}\lambda_{1,0} m_{1,t} e^{-\rho t} c_1(t)^{-\gamma} x(t)^{-(\gamma-1)} &= \lambda_{2,0} m_{2,t} e^{-\rho t} c_2(t)^{-\gamma} x(t)^{-(\gamma-1)} \\ c_1(t) &= \frac{m_1(t)^{\frac{1}{\gamma}}}{m_1(t)^{\frac{1}{\gamma}} + m_2(t)^{\frac{1}{\gamma}}} \epsilon(t)\end{aligned}$$

- consumption sharing rule $\rightarrow$ SDF

$$\begin{aligned}\xi(t) &= m_k(t) c_k(t)^{-\gamma} x(t)^{-(\gamma-1)} \\ &= \left(\frac{\epsilon(t)}{x(t)}\right)^{-(\gamma-1)} \epsilon(t)^{-1} \underbrace{\left(m_1(t)^{\frac{1}{\gamma}} + m_2(t)^{\frac{1}{\gamma}}\right)^{\gamma}}_{\text{diff. in beliefs abt inflation}}\end{aligned}$$

  - $\gamma = 1$ (EIS$=1$, sub. and income effects cancel), $B(t,T)$ not impacted by consumption risk sharing, differences in beliefs don't matter (Yan & Xiong, 2010)
  - $\gamma > 1$ (EIS<1, income effect dominates), $B(t,T)$ impacted by consumption risk sharing, (this paper)

Mechanics

- price of real bond

$$B(t,T) = E_t\left[\left(\frac{\epsilon(T)}{x(T)}\right)^{-(\gamma-1)}\left(\frac{\epsilon(t)}{x(t)}\right)^{(\gamma-1)}\frac{\epsilon(T)^{-1}}{\epsilon(t)^{-1}}\left(\frac{m_1(T)^{\frac{1}{\gamma}}+m_2(T)^{\frac{1}{\gamma}}}{m_1(t)^{\frac{1}{\gamma}}+m_2(t)^{\frac{1}{\gamma}}}\right)^{\gamma}\right]$$

- Inflation shocks independent of output shocks

$$B(t,T) = \underbrace{E_t\left[\left(\frac{\epsilon(T)}{x(T)}\right)^{-(\gamma-1)}\left(\frac{\epsilon(t)}{x(t)}\right)^{(\gamma-1)}\frac{\epsilon(T)^{-1}}{\epsilon(t)^{-1}}\right]}_{\text{indep. of diff. in beliefs}}\underbrace{E_t\left[\left(\frac{m_1(T)^{\frac{1}{\gamma}}+m_2(T)^{\frac{1}{\gamma}}}{m_1(t)^{\frac{1}{\gamma}}+m_2(t)^{\frac{1}{\gamma}}}\right)^{\gamma}\right]}_{\text{dep. on diff. in beliefs}}$$

- $\gamma = 1$ (Yan & Xiong, 2010), income and sub. effects cancel

$$E_t\left[\left(\frac{m_1(T)^{\frac{1}{\gamma}}+m_2(T)^{\frac{1}{\gamma}}}{m_1(t)^{\frac{1}{\gamma}}+m_2(t)^{\frac{1}{\gamma}}}\right)^{\gamma}\right] = E_t\left[\underbrace{\left(\frac{m_1(T)+m_2(T)}{m_1(t)+m_2(t)}\right)}_{\text{sum of 2 martingales}}\right] = 1$$

- $\gamma > 1$ (this paper), income effect dominates

Question: Nominal Rigidities

What makes inflation matter?

1. Traditional nominal rigidities in macro
   - Sticky prices and sticky wages
2. New nominal rigidities in macro
   - Sticky corporate leverage [Bhamra, Fisher & Kuehn (2011), Gomes, Jermann & Schmid (2013)]

How important is disagreement about inflation for macro-finance (via risk sharing channel)?

Inflation indexed bonds

A way to test whether disagreement about inflation really matters.

- In the model: inflation indexed bonds are a perfect hedge against inflation if you hold them to maturity, but earlier prices are not independent of inflation.
- Compute $\frac{\partial B(t,T)}{\partial \pi(t)}$ and $\frac{\partial B(t,T)}{\partial x(t)}$ to see how large they are.
- Do we see same signs and magnitudes empirically?

Recap: this paper

- Lucas endowment economy
- Agents with CRRA+multiplicative external habit and different beliefs about inflation
- Asset-Pricing Application: term structure of interest rates

There is more to asset pricing than term structure of bonds.
There is more to household heterogeneity than differences in beliefs.

Asset-Pricing

1. Real risk-free and Nominal risk-free Bonds
   - Term structure
   - Vol, Inflation effects
2. Corporate Bonds
   - Credit Spread Puzzle, Leverage Puzzle,
   - Vol, Term structure
   - Cross-section
   - Inflation effects
3. Equities
   - Equity Risk Premium Spread Puzzle, Term structure
   - Sharpe Ratio Puzzle, Term Structure
   - Vol, Term structure
   - Predicability
   - Cross-section
   - Inflation effects
4. FX
   - UIP, Vol, ...

We would like one model which explains everything!
How well does the model in this paper do for equity return vol?

Equity return vol

- Disagreement about expected consumption growth and CRRA preferences: need $\gamma < 1$ (EIS> 1) to get equity return vol above dividend growth vol
- Is the same true with disagreement about inflation? Probably.
- With CRRA and differences in beliefs, cannot simultaneously match bond and equity market facts.
- Need to separate RRA and EIS (e.g. EZW), introduce heterogeneity in RRA, or add habits

This paper already has habits

- easy to simultaneously match equity return vol. and equity risk premium in addition to term structures of real and nominal bonds?
- What about time series predictability?

Equities: time series predictability

- Model: disagreement about expected endowment growth rate, no learning
- Look at correlation between the log price-dividend ratio, $\ln \frac{P_t^Y}{Y_t}$, and the realized excess return on the market j years later, R_{t+j}^Y, for $j \in \{1, 2, 3, 5, 7\}$.
- All returns are annualized, model-based results are estimated on the basis of 50,000 years of simulated data.

$$Corr_t \left(\ln \frac{P_t^Y}{Y_t}, R_{t+j}^Y \right)$$

j (years)	1	2	3	5	7
Data	-0.18	-0.27	-0.08	-0.21	-0.05
Model:					
none	0.03	0.03	0.03	0.02	0.01
disagreement	0.03	0.03	0.03	0.02	0.01

Disagreement about expected endowment growth rate irrelevant for time series predictability of stock returns.

Equities: time series predictability II

Bhamra & Uppal (2013)

$$Corr_t\left(\ln \frac{P_t^Y}{Y_t}, R_{t+j}^Y\right)$$

j (years)	1	2	3	5	7
Data	-0.18	-0.27	-0.08	-0.21	-0.05
Model:					
none	0.03	0.03	0.03	0.02	0.01
disagreement	0.03	0.03	0.03	0.02	0.01
heterogeneity in RRA	-0.06	-0.06	-0.06	-0.05	-0.03

Disagreement about expected endowment growth rate irrelevant for time series predictability of stock returns.

Need to look at other forms of heterogeneity. [Chan & Kogan (2002)]

Conclusion

- In data: real yields impacted by differences in beliefs about inflation
- Nice theoretical result: real yields impacted by differences in beliefs about inflation even when inflation is independent of output. Driven by risk sharing across agents with different beliefs.
- How does disagreement about inflation compare with nominal rigidities?
- Can we do more than just look at risk-free bonds?
- Explore other forms of heterogeneity to match a wider set of asset pricing facts.