# Discussion: What is the Expected Return on a Stock?

**Harjoat S. Bhamra**

Imperial College Business School

2016

# A crude, but quick way to motivate what the paper is about

# Motivation

- Fundamental input into asset allocation models is expected return on a stock.
- Expected return is unobservable.
- If expected return assumed to be constant, need many years of data for accurate estimate
    - Doing it for the market is hard enough!
    - Merton (1980): 'even if the expected return on the market were known to be a constant for all time, it would take a very long history of returns to obtain an accurate estimate. And, of course, if this expected return is believed to be changing through time, then estimating these changes is still more difficult. '
- Wealth managers want accurate models of firm-level expected returns
- Existing approaches: CAPM with its beta, APT, linear factor models – need to estimate model parameters and then use it

Can we do better than this?

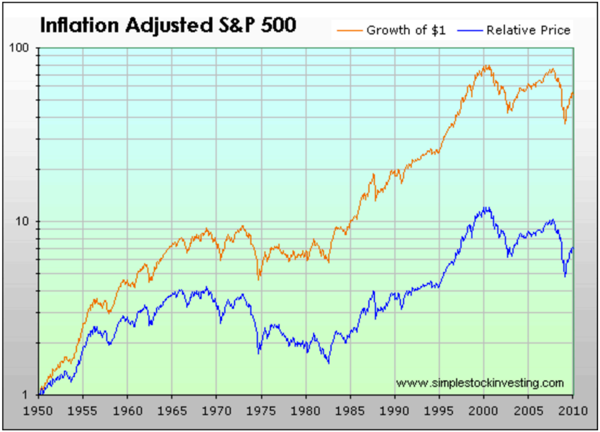



Figure: Orange – with dividends reinvested, Blue – without reinvesting dividends

# Questions

- Can we accurately estimate individual firm expected stock returns?
- How much money could we have made from our model? (Back-testing)
- How can we interpret whatever we have on the RHS of

$$E_t[R_{i,t+1}] = f(\textbf{something observable at date-}t, \textbf{fixed parameters}) \quad (1)$$

# How can we accurately estimate individual firm expected stock returns?

- Take your model and estimate it. Then go live!
- But there are problems!
- Goyal & Welch (2008) 'The evidence suggests that most models are unstable or even spurious. **Most models are no longer signicant even in sample (IS)**, and the few models that still are usually fail simple regression diagnostics. Most models have performed poorly for over 30 years IS. For many models, any earlier apparent statistical signicance was often based exclusively on years up to and especially on the years of the Oil Shock of 19731975. Most models have **poor out-of-sample (OOS) performance**, but not in a way that merely suggests lower power than IS tests. They predict poorly late in the sample, not early in the sample'
- Campbell & Thompson (2008) 'We show that simple restrictions on predictive regressions, suggested by investment theory, improve the out-of-sample performance of key forecasting variables and imply that investors could have **profited by using market timing strategies**.'

This paper: no parameters to estimate!

# Model I

- With no assumptions, prove that that expected return in excess of market given by

$$\frac{E_t[R_{i,t+1} - R_{m,t+1}]}{R_{f,t+1}} = \frac{1}{2}(SVIX_{i,t}^2 - \overline{SVIX}_{i,t}^2) + \Delta_{i,t} - \sum_i w_{i,t}\Delta_{i,t}, \qquad (2)$$

where

$$SVIX_{i,t} = var_t^*(R_{i,t+t}/R_{f,t+1}) \qquad (3)$$

$$\overline{SVIX}_{i,t}^2 = \sum_i w_{i,t} SVIX_{i,t}^2 \qquad (4)$$

$$\Delta_{i,t} = -\frac{1}{2} var_t^*[(R_{i,t+1} - R_{g,t+1})/R_{f,t+1}] \qquad (5)$$

# Model II

- Make assumptions

1.
$$\frac{E_t[R_{m,t+1} - R_{f,t+1}]}{R_{f,t+1}} = SVIX_t^2, \tag{6}$$

where

$$SVIX_t = var_t^*(R_{m,t+t}/R_{f,t+1}) \tag{7}$$

2. $\Delta_{i,t}$ independent of $i$ (depends on a common factor, fixed effects constant across stocks)

- Obtain a model, where RHS depends solely on observables from current option prices.

$$\boxed{\frac{E_t[R_{i,t+1} - R_{m,t+1}]}{R_{f,t+1}} = SVIX_t^2 + \frac{1}{2}(SVIX_{i,t}^2 - \overline{SVIX}_{i,t}^2)} \tag{8}$$

- No need to estimate any parameters!

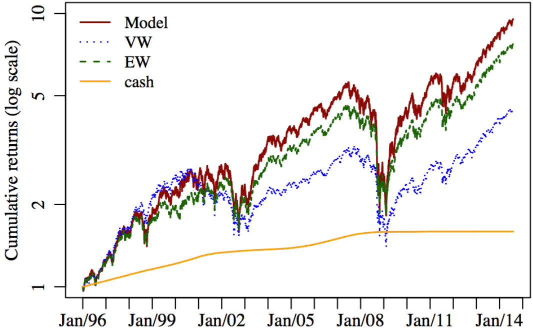
# How well does the model do relative to $\frac{1}{N}$?

- Portfolio weights

$$w_{i,t}^{XS}(\theta) = \frac{\text{rank}[E_t[R_{i,t+1}]]^{\theta}}{\sum_i \text{rank}[E_t[R_{i,t+1}]]^{\theta}} \quad (9)$$

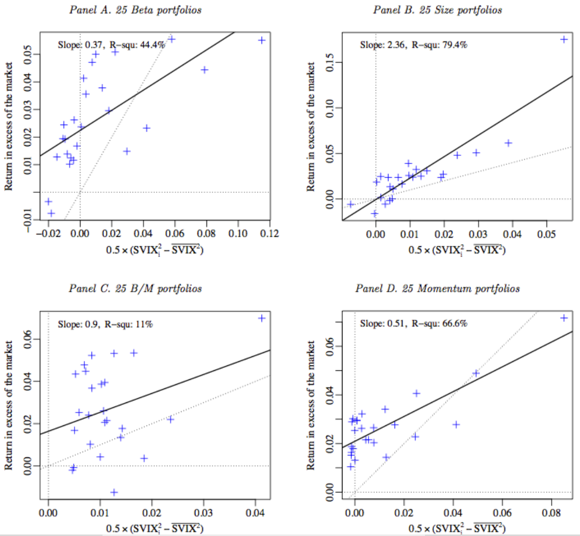
- Does slightly better than $1/N$ (special case of $\theta = 0$). Investors willing to pay performance fee of 0.27% to 1.38% per year.

*Panel A. One-month horizon*



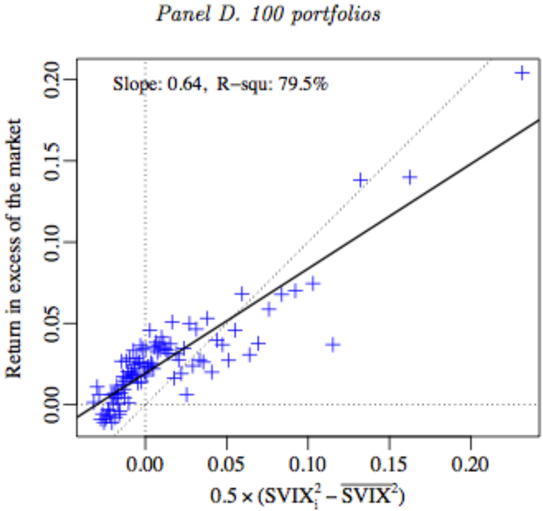
# Understanding the model I

- RHS seems to be strongly related to size and momentum, some link with beta, but not much with value



# Understanding the model II

- RHS seems to be strongly related to idiosyncratic vol



# The data: 1996-2014

- Focus on data from 1996 till 2014 means key events trigger the results: internet bubble and credit crunch. Can you see how well your model does over earlier periods?
- Authors do carry out principal components analysis, which suggests that extreme events drive results
- Perhaps if you use this model, you want to pray for disaster!
- But what if there aren't any? Will performance relative to existing benchmarks suffer?

# Transaction costs

- Investors willing to pay performance fee of 0.27% to 1.38% per year. In the UK common to pay 0.50% for using a platform (no real advice).
- What is left after transaction costs? $-0.23\%$ to 0.88% per year?

# Risk of Large Losses

- Loading up heavily on stocks with high $SVIX_{i,t}$ could be risky!
- Was it obvious Citibank etc. going to recover and that Fed and Government were going to be so pro-active?
- Need to quantity risk of large losses relative to $1/N$ etc.

# Long-short portfolio

- What if you borrow money to invest? How well does the model do relative to $1/N$ etc.
- Do you need to invest heavily when the rate at which you can borrow spikes?

# Tell us more about for which stocks the model does poorly

- Tell us more in terms of economics about when the assumption that $\Delta_{i,t}$ is independent of $i$ is poor
- This may give clues about how to improve performance relative to $1/N$

# Small growth stocks and other portfolios

- How well do you do in pricing small growth stocks relative to Fama and French (2015)? Bhamra & Shim (2016) suggest idiosyncratic failure risk is important.
- If you do well in a particular portfolio (for the cross-section), could you load up more on that portfolio to improve performance (in the time series)?

# Anomalies

- Can your approach give us any clues about idiosyncratic volatility and small-growth anomalies?
- I think it could do with more economic meat added to financial ketchup of existing model.
  - For example, does the emphasis on volatility point to the importance of real options models for the cross-section? Value/Growth and Size – Carlson, Fisher & Giammarino (2004), Idiosyncratic Volatility – Bhamra & Shim (2015).

# Conclusions

'Where there is doubt, may we bring faith.' [St Francis]

- What the paper does
  - Shifts debate away from 'Can we predict firm-level expected returns?' to 'How do we predict firm-level expected returns?'
  - Deals with annoying estimation issues by having no parameter estimation! This is very useful.
- What the paper does not do
  - Give us a way to make significantly more money than existing strategies (the $1/N$ issue)
  - But perhaps more work can bring hope?

# Derivations I

$$E_t[\ln \sum_i g_i R_{i,t}] \quad (10)$$

s.t.

$$\sum_i g_i = 1 \quad (11)$$

$$\mathcal{L} = E_t[\ln \sum_i g_i R_{i,t}] - \psi(\sum_i g_i = 1) \quad (12)$$

$$\frac{\partial \mathcal{L}}{\partial g_j} = E_t\left[\left(\sum_i g_i R_{i,t}\right)^{-1} R_{j,t}\right] - \psi = 0 \quad (13)$$

Muliplying through by $g_j$ and summing over $j$ reveals $\psi = 1$ and so

$$E_t\left[\left(\sum_i g_i R_{i,t}\right)^{-1} R_{j,t}\right] = 1. \quad (14)$$

# Derivations II

Therefore $\left(\sum_i g_i R_{i,t}\right)^{-1}$ is an SDF. This is the reciprocal of the so-called growth optimal portfolio return. Define

$$R_{g,t} = \sum_i g_i R_{i,t}, \tag{15}$$

where

$$\forall j,\ E_t\left[\left(\sum_i g_i R_{i,t}\right)^{-1} R_{j,t}\right] = 1. \tag{16}$$

Can we link this to entropy? Suppose we have set of A-D securities. Discretize state space. We have $K$ states and $N \leq K$ A-D securities.

$$P_{AD,k,t} = \begin{cases} 1 & \omega_{t+1} = \omega_{t+1}(k) \\ 0 & \omega_{t+1} \neq \omega_{t+1}(k) \end{cases} \tag{17}$$

$$k \in \{1, \ldots, N\} \tag{18}$$

and $N \leq K$

# Derivations III

$$R_{AD,k,t+1} = \begin{cases} \frac{1}{P_{AD,k,t}} & \omega_{t+1} = \omega_{t+1}(k) \\ 0 & \omega_{t+1} \neq \omega_{t+1}(k) \end{cases} \tag{19}$$

Now suppose we have a risk-neutral measure $q_1, \ldots, q_K$

$$P_{AD,k,t} = \frac{1}{R_{f,t+1}} q_k \tag{20}$$

Therefore

$$R_{AD,k,t+1} = \begin{cases} \frac{R_{f,t+1}}{q_k} & \omega_{t+1} = \omega_{t+1}(k) \\ 0 & \omega_{t+1} \neq \omega_{t+1}(k) \end{cases} \tag{21}$$

So why don't we maximize $E_t[\ln(\sum_{k=1}^{N} g_k R_{AD,k,t+1})]$ which is equivalant to maximizing

$$E_t[\ln(\sum_{k=1}^{N}(g_k/q_k)I_{\{\omega_{t+1}=\omega_k\}})] = \sum_{k=1}^{N} p_k \ln(g_k/q_k) \tag{22}$$

# Derivations IV

FOC

$$E_t\left[\frac{R_{AD,j,t+1}}{\sum_{k=1}^{N} g_k R_{AD,k,t+1}}\right] = \psi \tag{23}$$

or

$$p_k/g_k = \epsilon \tag{24}$$

Sum over states where A-D sec is traded

$$\sum_{k=1}^{N} p_k = \epsilon \sum_{k=1}^{N} g_k \tag{25}$$

$$\sum_{k=1}^{N} p_k = \epsilon \tag{26}$$

$N$ may be less than $K$, and so $\sum_{k=1}^{N} p_k$ may not be 1. Hence

$$g_k = \frac{p_k}{\sum_{k=1}^{N} p_k} \tag{27}$$

Interestingly, we then have

# Derivations V

$$E_t[\ln(\sum_{k=1}^{N} g_k R_{AD,k,t+1})] \tag{28}$$

$$= \ln R_{f,t+1} + \sum_{k=1}^{N} p_k \ln(g_k/q_k) \tag{29}$$

$$= \ln R_{f,t+1} + \sum_{k=1}^{N} p_k \ln(p_k/q_k) - \ln \sum_{k=1}^{N} p_k \tag{30}$$

$$= \ln R_{f,t+1} + D_{KL}(P|Q) - \sum_{k=N+1}^{K} p_k \ln(p_k/q_k) - \ln \sum_{k=1}^{N} p_k \tag{31}$$

$$= -E_t[\ln \Lambda_{t+1}] = -E_t\left[\ln \frac{M_{t+1}}{R_{f,t+1}}\right], \tag{32}$$

where

$$D_{KL}(P|Q) = \sum_{k=1}^{K} p_k \ln(p_k/q_k) \tag{33}$$

# Derivations VI

$$M_{t+1}(\omega_{t+1} = \omega_k) = \frac{Q_k}{P_k} \tag{34}$$

Hence

$$E_t[\ln M_{t+1}] = -D_{KL}(P|Q). \tag{35}$$

From Jensen

$$E_t[\ln M_{t+1}] \leq \ln E_t[M_{t+1}] = \ln 1 = 0 \tag{36}$$

Therefore, we obtain the rather well known Gibb's Inequality.

$$D_{KL}(P|Q) \geq 0 \tag{37}$$

The proof now has the attractive feature that we can see what the random variable $\frac{Q_k}{P_k}$ means. What if we start from an SDF. We should be able to write any given SDF as the reciprocal of the return on a portfolio of AD securities where the weights are the physical probabilities.

$$\Lambda_{t+1} = \frac{M_{t+1}}{R_{f,t+1}}, \tag{38}$$

where

$$M_{t+1}(\omega) = \frac{Q_{t+1}(\omega)}{P_{t+1}(\omega)}. \tag{39}$$

# Derivations VII

Hence

$$\Lambda_{t+1} = Q_{t+1}(\omega)\left(\frac{P_{t+1}(\omega)}{R_{f,t+1}}\right)^{-1} \tag{40}$$

$$\ln \Lambda_{t+1} = \ln\left[\frac{Q_{t+1}(\omega)}{P_{t+1}(\omega)}\right] - \ln R_{f,t+1} \tag{41}$$

Hence

$$E_t[\ln \Lambda_{t+1}] = E_t\left[\ln\left(\frac{Q_{t+1}(\omega)}{P_{t+1}(\omega)}\right)\right] - \ln R_{f,t+1} \tag{42}$$

$$= -D_{KL}(P|Q) - \ln R_{f,t+1}. \tag{43}$$

Define

$$\Lambda_{t+1} = \frac{1}{\sum_{k=1}^{N}} \tag{44}$$