# Network Centrality and the Cross Section of Stock Returns
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# Outline

- Aim
- Why do we care?
- Suggestions

# Paper's aim:

- Study how **network effects** impact stock returns across firms
- Do more central, i.e. 'hub' industries earn higher stock returns?
  - Fords chief executive, Alan R. Mulally, said the prospect of a failure of G.M. would cascade through the entire domestic auto industry and put millions of jobs at risk. [NYT, December 2, 2008]

# Rephrasing the paper's aims with basic theory

- No arbitrage & complete markets $\Rightarrow \exists!$ strictly positive SDF , $M$
- Standard asset pricing equation

$$P_{i,t} = E_t[M_{t,t+1}(D_{i,t+t} + P_{i,t+t})] \tag{1}$$

- We are interested in returns: $R_{i,t+1} = \frac{D_{i,t+t}+P_{i,t+t}}{P_{i,t}}$

$$E_t[R_{i,t+1}] - R_{f,t,t+1} = -R_{f,t,t+1} Cov_t[M_{t,t+1} R_{i,t+1}] \tag{2}$$

$$= -R_{f,t,t+1}\, \rho_t[M_{t,t+1}, R_{i,t+1}] Var_t[M_{t,t+1}] Var_t[R_{i,t+1}] \tag{3}$$

- $M_{t,t+1}$ depends on aggregate variables
- Kenneth's question: does the centrality of a sector impact the covariance of its returns with the SDF?

# Does centrality matter? Kenneth's Answer

- Yes, centrality does matter.
- Industries more central in terms of the US inter-firm trade network have significantly higher stock returns than less centrally located industries
- Not related to previous features driving cross-sectional stock return variation (size, value, growth)

| | Centrality | | | | | | |
|---|---|---|---|---|---|---|---|
| | Low 1 | 2 | 3 | 4 | High 5 | 1–5 | *t*-statistic |
| **Panel A: Monthly Returns (%)** | | | | | | | |
| *Levered* | | | | | | | |
| Value weighted | 1.21 | 1.93 | 2.25 | 2.29 | 2.44 | −1.23** | (−2.24) |
| Firm-level equal weighted | 0.28 | 0.97 | 1.30 | 1.23 | 1.42 | −1.14*** | (−4.69) |
| Industry-level equal weighted | 0.10 | 0.68 | 0.95 | 0.85 | 1.22 | −1.12*** | (−4.14) |
| *Unlevered* | | | | | | | |
| Value weighted | 1.09 | 1.72 | 2.10 | 2.11 | 2.07 | −0.99* | (−1.84) |
| Firm-level equal weighted | 0.29 | 0.91 | 1.19 | 1.15 | 1.29 | −1.00*** | (−4.33) |
| Industry-level equal weighted | 0.16 | 0.66 | 0.84 | 0.74 | 1.04 | −0.88*** | (−3.56) |
| Number of Industries | 59 | 77 | 82 | 83 | 84 | | |
| **Panel B: Industry Characteristics** | | | | | | | |
| Centrality | 0.03 | 0.06 | 0.11 | 0.23 | 1.44 | −1.41*** | (−6.97) |
| Concentration of Customers | 0.74 | 0.72 | 0.72 | 0.67 | 0.67 | 0.08* | (1.95) |
| Concentration of Suppliers | 0.61 | 0.58 | 0.60 | 0.62 | 0.68 | −0.07*** | (−4.68) |
| Log(Industry Output) | 7.74 | 8.53 | 9.11 | 9.84 | 11.27 | −3.53*** | (−34.67) |
| Log(Industry Average Market Equity) | 12.33 | 12.65 | 13.05 | 13.25 | 13.82 | −1.48*** | (−6.41) |
| Log(Industry Median Market Equity) | 12.15 | 12.34 | 12.51 | 12.46 | 12.57 | −0.42* | (−1.89) |

# Reconnecting with basic theory

$$E_t[R_{i,t+1}] - R_{f,t,t+1} = -R_{f,t,t+1}\,\rho_t[M_{t,t+1}, R_{i,t+1}] Var_t[M_{t,t+1}] Var_t[R_{i,t+1}]$$

- Cross-sectional variation in $E_t[R_{i,t+1}] - R_{f,t,t+1}$ driven by cross-sectional variation in $\rho_t[M_{t,t+1}, R_{i,t+1}]$ or $Var_t[R_{i,t+1}]$.
- From the point of view of network effects and centrality, $\rho_t[M_{t,t+1}, R_{i,t+1}]$ should be the important part

# Why do we care?

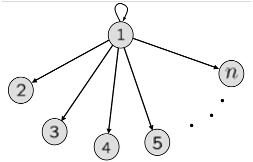
- Cross-sectional asset pricing focuses on relating differences in stock returns to characteristics, e.g. size/value/growth
- Cross-sectional puzzles in asset pricing should ultimately be related to sector/industry/firm specific characteristics.
  - Ken's work can be seen as an example of this, focusing on the economically intuitive idea of industry centrality within a network
- Eventually use understanding of cross-sectional risk premia to assess policy implications on a sector by sector basis.
  - The US car industry was bailed out, because of its perceived centrality
  - Would an increase in expected stock returns for the car industry increase expected aggregate stock returns?
  - What about the welfare implications?

# Suggestions

- More data (network matrix based on 1 year of data). Obvious point.
- Explain centrality measure more clearly. Key concept: I did not get much intuition from the paper.
- Directly test a theory stated mathematically rather than a qualitative hypothesis.
  - Existing theory provides conditions under which idiosyncratic shocks can have an aggregate impact [Acemoglu, Carvalho, Ozdaglar, and Alireza Tahbaz-Salehi, (2012), The network origins of aggregate fluctuations, Econometrica] Are these conditions satisfied empirically?
  - Test an asset pricing theory based on networks of input/output flows between sectors – develop one! (hard). See Buraschi & Trojani (2012).
- Risk premia are driven by network changes

# Centrality I

Think about economy with one sector that provides inputs to itself and all the other sectors.




- Sector 1 is very central.
- Write network as matrix (n=4).

$$\begin{pmatrix} x_{11} & 0 & 0 & 0 \\ x_{21} & 0 & 0 & 0 \\ x_{31} & 0 & 0 & 0 \\ x_{41} & 0 & 0 & 0 \end{pmatrix}$$

- Sector $i$ receives $x_{ij}$ from Sector $j$
- Normalize matrix so elements of columns sum to 1. Get a new matrix $W$=(w$_{ij}$), where $w_{ij} = \frac{x_{ij}}{\sum_{i=1}^{4} x_{ij}}$

$$\begin{pmatrix} w_{11} & 0 & 0 & 0 \\ w_{21} & 0 & 0 & 0 \\ w_{31} & 0 & 0 & 0 \\ w_{41} & 0 & 0 & 0 \end{pmatrix}$$

# Centrality II

- What can we do with $W$?
  - We can see how a vector of inputs $x = (x_1, \ldots, x_n)^T$ is distributed across the 4 sectors via $Wx$, e.g.

$$\begin{pmatrix} w_{11} & 0 & 0 & 0 \\ w_{21} & 0 & 0 & 0 \\ w_{31} & 0 & 0 & 0 \\ w_{41} & 0 & 0 & 0 \end{pmatrix} (1,0,0,0)^T = (w_{11}, w_{21}, w_{31}, w_{41})^T$$

  - An input produced by sector 1 gets transferred to all 4 sectors.
  - What about input produced by sector 2?

$$\begin{pmatrix} w_{11} & 0 & 0 & 0 \\ w_{21} & 0 & 0 & 0 \\ w_{31} & 0 & 0 & 0 \\ w_{41} & 0 & 0 & 0 \end{pmatrix} (0,1,0,0)^T = (0,0,0,0)^T$$

  - It does not go anywhere. Sector 2 is at the edge of the trade network.

# Centrality III

- After many rounds of intersectoral trade a vector of inputs $x$ will get mapped into a new vector.
- After $k$ rounds of trade

$$W^k x = \text{new input vector} \tag{4}$$

- If Sector $j$ is more central, it will have more input passing through it: the $j$'th element of the new input vector will be larger.

- So what has $W$ got to do with which sectors are more central than others?
- Where do the eigenvectors of $W$ tie in?
- Use eigenvectors to compute the new vector $W^k x$

# Centrality III

- $W$ has eigenvectors $e_i$, $i \in \{1, \ldots, 4\}$ with corresponding eigenvalues $\lambda_i$, $i \in \{1, \ldots, 4\}$
$$We_i = \lambda_i e_i \tag{5}$$
- Any vector of inputs $x$ can be written wrt to the basis of eigenvectors of $W$ as
$$x = \sum_{i=1}^{4} \overbrace{b_i}^{\text{scalar}} \times \overbrace{e_i}^{\text{eigenvector}} \tag{6}$$
- After one round of intersectoral trade, input is transferred as follows
$$Wx = \sum_{i=1}^{4} b_i \underbrace{\overbrace{\lambda_i}^{\text{eigenvalue}} \quad e_i}_{We_i = \lambda_i e_i}$$
- After $k$ rounds of intersectoral trade, initial input is transferred as follows
$$W^k x = \sum_{i=1}^{4} b_n \lambda_n^k e_n$$
- Most of the eigenvalues make negligible contribution to the final vector. What matters is the contribution from largest (dominant) eigenvalue ($i = d$)
$$W^k x \approx b_d \lambda_d^k e_d = b_d \lambda_d^k (e_{d,1}, \ldots, e_{d,4})^T \text{ [Perron-Frobenius Theory]}$$
- $e_{d,i}$ is the $i$'th element of principal eigenvector of $W$: measures the centrality of Sector $i$

# Existing theory: adding up risks

Acemoglu et al (micro foundations for macro risk):

$$Var\left[\sum_{i=1}^{n} v_i\tilde{\epsilon}_i\right] \sim \left(\frac{1}{\sqrt{n}} + \overbrace{\text{dominant sector effect} + \text{interconnectivity effect}}^{\text{these terms lead to idio risk having an aggregate impact}}\right) \tag{7}$$

- dominant sector effect: only a small fraction of sectors are responsible for the majority of the input supplies in the economy.
    - shocks to dominant sectors propagate through the entire economy as their low productivity leads to lower production for all of their downstream sectors
- interconnectivity effect
    - A group of sectors has common suppliers. A shock to the the suppliers propagates through the sectors connected to them.
- Relate the dominant sector effect and the connectivity effect to the properties of the trade matrix.
    - Lots of work on this: algebraic graph theory applied to weighted digraphs
- Are this effects quantitatively important? Can you distinguish between them?

# (Pseudo) theory

- Variance is not enough. Risk premia are driven by covariances

$$E_t[R_{i,t+1}] - R_{f,t,t+1} = -R_{f,t,t+1}\, \rho_t[M_{t,t+1}, R_{i,t+1}] Var_t[M_{t,t+1}] Var_t[R_{i,t+1}] \tag{8}$$

- Special case: Extension of CAPM

$$M_{t,t+1} = A + F_1 R^W_{t+1} + \sum_{j=1}^{K} F_j Z_{j,t+1}, \tag{9}$$

where

$$R_{W,t+1} = \sum_{i=1}^{I} \frac{P_{i,t}}{W_t} R_{i,t+1} \tag{10}$$

$Z_{j,t+1}$ other factors

- Compute $Cov_t[\sum_{i=1}^{I} \frac{P_{i,t}}{W_t} R_{i,t+1}, R_{i,t+1}]$ using the algebra in Acemoglu et al.
  - Identify the extra terms which arise due to network effects. How large are they?
  - Do the same for other factors, in particular sector size.

# Final comment

- Asset pricing theory links conditional expected returns to second moments of changes (in prices, dividends, consumption, wealth, wealth distribution)
- Data on centrality focuses on a snapshot of a input/output matrix.
- The risk of a change in network structure is what is priced; a static network where the is no chance of sectors gaining/losing centrality will not generate risk premia
- What we really need to know is how the principal eigenvector of the input/output matrix changes
  - We do know some things about eigenvector stability. See Terrance Tao's blog. (http://terrytao.wordpress.com/2008/10/28/when-are-eigenvalues-stable/)
  - The more stable prinicpal eigenvector is wrt network changes, the smaller risk premia will be
- Theorists: go and study how eigenvectors change
- Empiricists: go and study how to measure eigenvector changes

# Conclusion

- Very exciting and potentially v. useful research topic.
- Hopefully Kenneth's centrality will be very high in the near future.