

Generalized Risk Premia by Paul Schneider

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2014

Outline

- Aim
- Why do we care?
- Model Summary & Results
- Comments

My View of Paper's Aim

- **Main question:** What can we learn about the SDF from option pricing data and the VIX index (in addition to using SP500 index returns)?
- Use data on option prices, VIX and SP500
- Look at 2 different models
 - Extension of long-run risk model: Drechsler & Yaron (2011)
 - Kuechler & Tappe (2011): Levy time series model for SP500 index returns
- What does the orthogonal expansion of the martingale component of SDF look like across models?

$$\Lambda_T = \underbrace{E_0[\Lambda_T]}_{\text{time-adjustment}} \underbrace{\frac{\Lambda_T}{E_0[\Lambda_T]}}_{\text{risk-adjustment (martingale)}} \quad (1)$$

- Expansion in basis of orthogonal polynomials – decomposition of risk-adjustment factor in SDF

$$\frac{\Lambda_T}{E_0[\Lambda_T]} = 1 + \sum_{j=1}^{\infty} c_j \underbrace{H_j(R_{0,T})}_{\text{poly. of degree } j}$$

- Non-linear decomposition (going beyond simple linear factor models)
- Different models: different c'_j s
- Different c'_j s: different risk premia

$$E_0[X_T] = CEQ_0[X_T] - Cov_0 \left(\frac{\Lambda_T}{E_0[\Lambda_T]}, X_T \right)$$

- What do options tells us about higher order risk premia? How do estimates differ across models?

Understanding asset prices

Most variation in asset returns driven by changes in expected risk premia.

- In portfolio allocation need to understand expected risk premia.
 - Use factor models to estimate and decompose risk premia.
 - E.g. suppose you want to insure against crash risk: can use factor model to find assets which have a crash risk premium
- Suppose want to understand benefits of diversifying across industries.
 - Do different industries have risk premia exposed to different factors?
- Suppose some hedge fund claims to have a high alpha strategy
 - They are probably counting exposure to some risk factor as alpha
 - A good factor model can help us gain a better understanding of the hedge funds performance.

But where do these factor models come from? Are the standard ones any good?

Going beyond linear factor models

- Typical linear factor model (e.g. CAPM model and the Fama-French 3 factor model)

$$\frac{\Lambda_T}{E_0[\Lambda_T]} = \sum_j a_j F_j \quad (2)$$

- CAPM model and the Fama-French 3 factors cannot explain risk premia on variance swaps. (Carr and Wu (2007))
- Option prices are nonlinear. Option price data suggests that **nonlinear** factors such as variance are priced. Also skewness.
- This paper provides a general nonlinear factor model as an orthogonal expansion of the martingale cpt of the SDF
- Expansion gives a way of generating risk premia on swaps written on any moment: variance and skewness are special cases.
Potentially help improve our understanding of expected risk premia

The Expansion & the SDF I

Definition 1

A stochastic process $\Lambda = (\Lambda_t)_{t \in \mathcal{T}}$ is an SDF process if

- $\Lambda_0 = 1$
- $\forall t \in \mathcal{T}, \Lambda_t > 0$, with probability 1
- ΛS_i is a local martingale, where S_i is the price of asset i

Example 2

$$\Lambda_t = \underbrace{e^{-rt}}_{\text{time adj. fac.}} \underbrace{e^{-\frac{1}{2}\Theta^2 - \Theta Z_t}}_{\text{risk adj. fac. - martingale}} \quad (3)$$

where

$$\Theta = \frac{\mu - r}{\sigma}. \quad (4)$$

More generally

$$\Lambda_t = \underbrace{E_0[\Lambda_t]}_{\text{time-adjustment}} \underbrace{\frac{\Lambda_t}{E_0[\Lambda_t]}}_{\text{risk-adjustment (martingale)}} \quad (5)$$

The Expansion & the SDF II

- SDF decomposition

$$\Lambda_t = \underbrace{E_0[\Lambda_t]}_{\text{time-adjustment}} \underbrace{\frac{\Lambda_t}{E_0[\Lambda_t]}}_{\text{risk-adjustment (martingale)}} \quad (6)$$

- Martingale $M_t = \frac{\Lambda_t}{E_0[\Lambda_t]}$
- The martingale used for risk pricing is the Radon-Nikodym derivative, $\frac{d\mathbb{P}}{d\mathbb{Q}}$, or likelihood ratio, \mathcal{L} .

The Expansion

- Orthogonal expansion of martingale for risk pricing

$$M_t = 1 + \sum_j c_j \cdot H_j(R_{0,t}) \quad (7)$$

- Could perform a non-orthogonal expansion

$$M_t = 1 + \sum_j a_j \cdot R_{0,t}^j \quad (8)$$

- Different models \rightarrow different c_j 's \rightarrow different expected risk premia

Payoff dependent on likelihood ratio

- Underlying index value at date- t , S_t
- Price of forward on index at date- t , $F_{t,T}$
- Return:

$$R_{0,T} = \ln \frac{S_T}{F_{t,T}} \quad (9)$$

- Payoff based on \mathcal{L}

$$\mathcal{L}(R_{0,T}) = E^{\mathbb{Q}_T}[\mathcal{L}|R_{0,T}] \quad (10)$$

- Payoff of likelihood ratio swap (martingale swap or risk pricing swap)

$$\underbrace{\mathcal{L}(R_{0,T})}_{\text{underlying at date-T}} - \underbrace{E_0^{\mathbb{Q}_T}[\mathcal{L}(R_{0,T})]}_{\text{forward price, i.e. CEQ of underlying at date-T}} \quad (11)$$

The Swap and the Expansion

Likelihood ratio swap + expansion + basic asset pricing equation \rightarrow write risk premium on likelihood ratio swap as a sum of risk premia on swaps on moments of returns

$$RP_{0,T}^{\mathcal{L}} = \sum_j a_j \cdot RP_{0,T}^j \quad (12)$$

Two Models

- Drechsler & Yaron
 - Consumption-based asset pricing model
 - EZW rep. agent
- Kuechler & Tappe (extension of Black & Scholes)
 - Exogenous process for SP500 index
- For each model
 - Find dynamic portfolio (options and forwards) which replicates likelihood ratio swap

Main results

- Excess returns from the two models closer during crises periods
- Negatively correlated during normal times

Comments I

- Why not use log variables for expansion
 - Expand $\ln M_T$

$$\ln M_T = \sum_j c_j \cdot H_j(R_{0,T}) \quad (13)$$

- Define an entropy swap based on function

$$E^{\mathbb{Q}_T}[\ln M_T | R_{0,T}] \quad (14)$$

- Relationship with Martin (2013) and Backus, Chernov & Martin (2012)?

Comments II

- Flight to quality in crises suggests looking at heterogeneous agent models
 - What is the trading strategy for a consumption-based model, where agents have differences in risk aversion and or beliefs?
 - More interesting SDF
 - Gets past the annoying feature of no trade in zero net supply assets in rep. agent economies.

Conclusion

- Interesting toolkit
- Cleaner expansions (log variables)?
- Look at economies where agents trade with each other