Risk-Adjusted Capital Allocation and Misallocation

Discussion

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Aims

- A cross-sectional macro-finance paper

- Connect variations in the marginal productivity of capital ($MP\underbrace{K}_{\text{das Kapital}}$) to variations in expected risk premia

Why do we care?

- Cross-sectional puzzles in asset pricing should ultimately be related to sector/industry/firm specific characteristics

- Eventually use understanding of cross-sectional risk premia to assess policy implications on a sector by sector basis.

Outline of Paper

- Build partial equilibrium model of firms
- *PV* of future *MPK* equal across firms. Don't ignore differences between $\mathbb{P}$ and $\mathbb{Q}$
- Firm-level operating profits are decreasing with the stochastic price of risk – extent of this loading is different across firms.
- Generates heterogeneity in both *MPK* and expected risk premia.
- Use model to infer heterogeneity in *MPK* from heterogeneity in risk premia.
- Compare inferred heterogeneity in *MPK* with direct measure – how close are they?

Model

- Cross-section of firms. Firm i pays out dividend flow $D_{i,t}$

$$(1) \quad D_{i,t} + I_{i,t} = \Pi_{i,t}$$

- Bellman equation

$$(2) \quad V_{i,t} = \sup_{K_{i,t+1}} D_{i,t} + E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} V_{i,t+1} \right]$$

where

$$(3) \quad K_{i,t+1} = I_{i,t} + (1-\delta) K_{i,t}$$

- FOC

$$(4) \quad E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} [\Psi_{i,t+1} + (1-\delta)] \right] = 1,$$

where

$$(5) \quad \Psi_{i,t+1} = \frac{\partial \Pi_{i,t+1}}{\partial K_{i,t+1}}$$

Interpreting the FOC

- Present value of future marginal product of capital is constant across firms

$$(6) \quad E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\Psi_{i,t+1}\right] \text{ is independent of } i$$

where

$$(7) \quad \Psi_{i,t+1} = \frac{\partial \Pi_{i,t+1}}{\partial K_{i,t+1}}$$

- Explicitly change measure from $\mathbb{P}$ to $\mathbb{Q}$

$$(8) \quad E_t^{\mathbb{Q}}\left[e^{-r_{f,t}}\Psi_{i,t+1}\right] \text{ is independent of } i \Rightarrow E_t^{\mathbb{Q}}\left[\Psi_{i,t+1}\right] \text{ is independent of } i$$

- Risk-neutral expected value of future marginal product of capital is identical across firms
- Interpret in two ways
    1. Old fashioned macro view
    2. Modern macro-finance view

Old fashioned macro view

- $\mathbb{P}$ and $\mathbb{Q}$ are same (risk premia are just noise) and so

$$(9) \quad E_t^{\mathbb{Q}}\left[\Psi_{i,t+1}\right] \text{ is independent of } i \Rightarrow E_t\left[\Psi_{i,t+1}\right] \text{ is independent of } i$$

- Then look at data and see that $E_t\left[\Psi_{i,t+1}\right]$ is not independent of i and deduce that there is misallocation
- Explore welfare implications of this misallocation.

Modern macro-finance view

- $\mathbb{P}$ and $\mathbb{Q}$ are not same and so

(10)

$$E_t^{\mathbb{Q}}\left[\Psi_{i,t+1}\right] \text{ is independent of } i \text{ but } E_t\left[\Psi_{i,t+1}\right] \text{ can depend on } i$$

- Then look at data and see that $E_t\left[\Psi_{i,t+1}\right]$ is not independent of i and deduce that there are risk premia instead of misallocation.
    - At the very least – misallocation must be less than the traditional macro view suggests.
- Empirical asset pricing – subset focuses on understanding cross-sectional differences in risk premia.
- Why not connect cross-sectional differences in $E_t\left[\Psi_{i,t+1}\right]$ to cross-sectional differences in risk premia?

Connecting marginal products of capital to risk premia I

- Need structure on $\Pi_{i,t}$ and SDF Λ to exploit FOC
- Assume $\pi_{i,t} = \ln \Pi_{i,t}$ and SDF $\lambda_t = \ln \Lambda_t$ are linear functions of Gaussian rv's.

$$(11)\quad E_t^{\mathbb{Q}}\left[e^{-r_{f,t}}[\Psi_{i,t+1} + (1-\delta)]\right] = 1 \Rightarrow \ln E_t^{\mathbb{Q}}[\Psi_{i,t+1}] \approx r_{f,t} + \delta$$

$$(12)\quad \boxed{E_t^{\mathbb{Q}}[\psi_{i,t+1}] \approx r_{f,t} + \delta - \frac{1}{2} Var_t^{\mathbb{Q}}[\psi_{i,t+1}]}$$

- Use above equation to derive $k_{i,t+1}$ – will depend on risk-neutral expectations
- Marginal product of capital will depend on risk-neutral expectations

Connecting marginal products of capital to risk premia II

- Assumptions on operating profits and SDF

(13) $\Pi_{i,t} = Ge^{\beta_i x_t + z_{i,t} + \theta k_{i,t}} \Rightarrow \psi_{i,t} = g + \beta_i x_t + z_{i,t} - (1-\theta)k_{i,t}$

(14) $\lambda_{t+1} - \lambda_t = E_t[\lambda_{t+1} - \lambda_t] - (\gamma_0 + \gamma_1 x_t)\sigma_\lambda \epsilon_{t+1}$

where

(15) $x_{t+1} = \rho x_t + \sigma_x \epsilon_{t+1}$

- What does this mean?
  - $(\gamma_0 + \gamma_1 x_t)\sigma_\lambda$ is the price of risk, $\gamma_1 < 0$ – low x means higher price of risk
  - ϵ is part of what describes the aggregate state – unexpected improvement in ϵ leads to negative shock to SDF and increase in x
  - Operating profits load on the price of risk and not on ϵ

Connecting marginal products of capital to risk premia III

- FOC leads to

$$k_{i,t+1} = g + \beta_i \frac{E_t^{\mathbb{Q}}[x_{t+1}]}{1-\theta} + E_t[z_{i,t+1}] - (r_{f,t} + \delta) \tag{16}$$

- Plausible that have high price of risk (low x) in bad aggregate states. $E_t^{\mathbb{Q}}[x_{t+1}] = \rho x_t - (\gamma_0 + \gamma_1 x_t)\sigma_\lambda \sigma_x = \widehat{\rho} x_t - \widehat{\gamma}_0$.

$$k_{i,t+1} \propto \beta_i \frac{\widehat{\rho} x_t}{1-\theta} + E_t[z_{i,t+1}] \tag{17}$$

- $k_{i,t+1}$ lower when risk is priced more severely – real investment lower in bad aggregate states
  - if real interest rate is lower in bad states (higher precautionary savings demand in bad states or basic intertemporal smoothing) this effect is reduced – good because real investment less volatile than price of risk

Connecting marginal products of capital to risk premia IV

- log marginal product of capital

$$(18)\qquad \psi_{i,t} \propto \left(1+\widehat{\rho}\frac{\theta}{1-\theta}\right)\beta_i x_t$$

$$(19)\quad E_t[\psi_{i,t+1}] \propto \left(1+\widehat{\rho}\frac{\theta}{1-\theta}\right)\beta_i \rho x_t$$

- Expected risk premium of firm i

$$(20)\quad \ln E_t[R^e_{i,t+1}] \propto \beta_i(\gamma_0+\gamma_1 x_t)$$

- Model connects expected risk premia to cross-section of marginal products of capital.

Cross-sectional variance of expected risk premia v. marginal products of capital

- Cross-sectional variance of expected risk premia is just under half of the cross-sectional variance of marginal products of capital

- The modern macro-finance view suggests there is less misallocation than the traditional macro view.

- Heterogeneity in δ, θ does not generate in enough variance in expected risk premia

Summary of Contributions

- Differences in exposure of operating profits to the price of risk can explain both cross-sectional variation in expected risk premia and MPK

- Differences in firm-level production parameters can explain some of the cross-sectional variation in MPK, but not expected risk premia,

Comments I

- Do operating profits really depend on the price of risk?
- Could you get this in general equilibrium – doubtful.

Comments II

- Consider a model with growth options – very common in cross-sectional asset pricing literature.
- Rise in fundamental volatility makes growth options more valuable – investment is delayed – will impact MPK. Will also impact expected risk premium.
- There is cross-sectional heterogeneity in the proportion of firm value derived from growth options versus assets in place.
- Heterogeneity in growth options/assets in place will generate differences in expected risk premia and MPK in a model with stochastic volatility.
- Try and build a ge model starting from Gomes, Kogan & Zhang (2003)?

Comments III

- In a model with no frictions, there is no misallocation.
- Does it really make sense to talk about reductions in TFP stemming from misallocation, when heterogeneity in MPK is an efficient outcome?
- If it's not misallocation, why call it misallocation?