Arbitrage-Free Bond Pricing With Dynamic Macroeconomic Models

Discussant: Harjoat S. Bhamra, UBC

Michael F. Gallmeyer **Burton Hollifield**

Francisco Palomino **Stanley E. Zin**

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The Bigger Picture

- There is interaction between
  1. Monetary Policy
  2. Real Economy
  3. Asset Prices
- Some research has focused on Monetary Policy and the Real Economy (Woodford, sticky price/wage models)

Asset Prices and Monetary Policy

- What about Asset Prices?
- When should Monetary Policy take Asset Prices into consideration?
  - Greenspan and asset price bubbles, Bernanke and the credit crisis
- If Monetary Policy does take Asset Prices into account, what are the real effects? What are the effects on Portfolio Choices and Asset Prices?
- Large research agenda: needs some focus!

What This Paper Does

- Makes inflation a consequence of monetary policy
- How is it done?
  1. Assumption about monetary policy
     - Nominal interest rate (nominal yield on 1 year bond) follows a Taylor Rule

$$i_t = -\ln b_t^{1,\$} = f(\text{real cons growth}, \text{cons growth vol}, p_t, \text{Exog Shocks})$$

  2. Introduce a real SDF (m_t)
     - Find the price level which makes nominal interest rate consistent with no-arbitrage
- Real SDF is not exogenous as in standard term structure models
  - Real SDF comes from representative agent's preferences and exogenous real consumption (Lucas)

Questions We Can Answer With This Paper's Model

- Can ask how the following affect inflation and the nominal yield curve:
  - Changing f (Taylor Rule)
  - Changing the rep agent's preferences
  - Statistical assumptions about real consumption growth

Issues Beyond The Model

- Can households and a monetary policy setting agent be aggregated into a rep agent?
- Monetary policy has no real implications: real consumption is exogenous
  - Impossible to assess welfare implications of monetary policy (see Palomino (2007))
- The only way monetary policy affects nominal yields is via inflation

How Does Inflation Affect Nominal Bond Prices?

- Price of a nominal bond
    - Date-t price of a bond which pays out one dollar at date $t+n$

$$b_t^{n,\$} = b_t^n E_t[P_t/P_{t+n}]\left[1 + Cov_t\left(\frac{m_t/m_{t+n}}{E_t[m_{t+n}/m_t]}, \frac{P_t/P_{t+n}}{E_t[P_t/P_{t+n}]}\right)\right]$$

- $p_t = \ln P_t,\ m_t = P_t m_t^{\$}$
- Expected inflation
- Covariance of inflation with real SDF- Inflation Risk Premium

The Model Framework: **Endogenous** Inflation Model

- Taylor Rule

$$i_t = -\ln b_t^{1,\$} = f(x_t, \text{cons growth vol}, p_t, \text{Exog Shocks})$$

But

$$b_t^{1,\$} = b_t^1 E_t[P_t/P_{t+1}]\left[1 + Cov_t\left(\frac{m_t/m_{t+1}}{E_t[m_{t+1}/m_t]}, \frac{P_t/P_{t+1}}{E_t[P_t/P_{t+1}]}\right)\right]$$

- Inflation is now endogenous!

Consumption Growth

- Consumption growth $x_{t+1} = c_{t+1}/c_t$ is mean-reverting with stochastic vol v_t

$$
\begin{aligned}
x_{t+1} &= (1-\phi_x)\theta_x + \phi_x x_t + v_t^{1/2}\epsilon_{t+1}^x \\
v_{t+1} &= (1-\phi_v)\theta_v + \phi_v v_t + \sigma_v \epsilon_{t+1}^v
\end{aligned}
$$

Representative Agent

- Real SDF (m) comes from a representative agent with Epstein-Zin preferences
    - RRA, γ
    - EIS, ψ
- Agent cares about path of future consumption: continuation utility normalized by its CEQ enters the SDF

$$m_{t+1} = \beta x_{t+1}^{-1/\psi} \left(\frac{U_{t+1}}{CEQ_t(U_t)} \right)^{1/\psi - \gamma}$$

$$u(CEQ_t(U_{t+1})) = E_t(u(U_{t+1})),$$

where

$$u(x) = \frac{x^{1-\gamma}}{1-\gamma}$$

Exogenous Monetary Policy & Endogenous Inflation

- Monetary policy is chosen exogenously (Taylor Rule)

$$i_t = -\ln b_t^1 = \bar{\tau} + \tau_x x_t + \tau_p p_t + s_t$$

  - Exogenous s_t, AR(1), orthogonal to shocks driving x_t and v_t
- Obtain endogenous inflation

$$p_t = \bar{\pi} + \pi_x x_t + \pi_v v_t + \pi_s s_t$$

What Does Endogenizing Inflation Buy Us?

- **Exogenous inflation**
- The exogenous inflation process in the paper does **not** covary with real SDF → **zero** inflation risk premium → downward sloping nominal term structure

$$b_t^{n,\$} = b_t^n E_t[P_t / P_{t+n}]$$

- **Endogenous inflation**
- Endogenous inflation process in the paper **does** covary with real SDF → **non-zero** inflation risk premium → upward sloping nominal term structure & more volatility at long end

$$b_t^{n,\$} = b_t^n E_t[P_t / P_{t+n}] \left[1 + Cov_t \left(\frac{m_t / m_{t+n}}{E_t[m_{t+n}/m_t]}, \frac{P_t / P_{t+n}}{E_t[P_t / P_{t+n}]} \right) \right]$$

Exogenous v Endogenous Inflation

- Could just set exogenous inflation equal to the endogenously derived inflation process and obtain identical nominal term structure (upward sloping and with more volatility at long end)
- But, can ask following questions:
  - How does choice of monetary policy affect inflation and nominal term structure?
  - How do EIS and RRA affect inflation and nominal term structure?
  - How does the stochastic vol assumption affect inflation and nominal term structure

How Does Monetary Policy Affect Inflation and Nominal Yield Curve?

- Taylor Rule

$$i_t = -\ln b_t^1 = \bar{\tau} + \tau_x x_t + \tau_p p_t + s_t$$

  - Exogenous s_t, AR(1), orthogonal to shocks driving x_t and v_t
- Implied inflation

$$p_t = \bar{\pi} + \pi_x x_t + \pi_v v_t + \pi_s s_t$$

  - Making nominal rate more sensitive to output growth (larger τ_x) $\rightarrow$ inflation more sensitive to stochastic vol (larger π_v) $\rightarrow$ nominal yield curve shifted down and more upward sloping & higher nominal yield volatility
  - Making nominal rate more sensitive to price level (larger τ_p) $\rightarrow$ inflation more sensitive to output growth (larger π_x), less sensitive to stochastic vol (smaller π_v) $\rightarrow$ nominal yield curve shifted down and more upward sloping $\rightarrow$ higher nominal yield volatility $\rightarrow$ nominal yield curve shifted down and less upward sloping & less nominal yield volatility

Intuition

- More intuition for Taylor → Implied inflation
- Existing intuition: Implied Inflation → nominal SPD → nominal yield curve

$$b_t^{n,\$} = b_t^n E_t[P_t/P_{t+n}]\left[1 + Cov_t\left(\frac{m_t/m_{t+n}}{E_t[m_{t+n}/m_t]}, \frac{P_t/P_{t+n}}{E_t[P_t/P_{t+n}]}\right)\right]$$

- Add extra step: Implied Inflation → nominal SPD → **inflation risk premium** → nominal yield curve

How Does EIS Affect Inflation and Nominal Yield Curve?

- This has **both** real and nominal effects!
- **Real effects**
- Higher EIS → less demand for long-term bonds, higher real long-term yields → real term structure less downward sloping & Δ of real yields wrt output growth is lower → less volatile real long-term yields [But higher EIS → lower real risk-free rate (Weil)?]
- **Nominal effects**
- Higher EIS → more upwards sloping nominal term structure % more volatile nominal long-term yields more demand for bonds, lower long-term yields

Questions/Comments

- Stochastic vol seems very important to get upwards sloping nominal term structure.
- Would be instructive to see model without stochastic vol in consumption growth
- For the same parameters that generate reasonable nominal term structure dynamics, can you get a reasonable equity risk premium?
- Link autocovariance of nominal SDF to term structure of inflation risk premium
- How different are the statistical properties of implied inflation from data?
- Can you classify the family of monetary policy rules that gives realistic implied inflation?

Extensions

- With existing model, look at
  - Equity Risk Premium
  - Bond Options and their Greeks (see how they respond to macro factors and Taylor Rule)
  - Corporate Bonds: how does monetary policy affect risk-neutral default probabilities?