Portfolio Choice with Model Misspecification

A Foundation for Alpha and Beta Portfolios

Discussion

Harjoat S. Bhamra

Imperial College Business School

23 August 2018

Aims (& some Notation) I

- How do you structure a static portfolio when
  - you don't know the correct factors – what they are and how many there are.
  - you don't know the correct model for expected risk premia, but you assume they are decomposed into factor-independent and factor-dependent components
  - there is no asymptotic arbitrage

Aims (& some Notation) II

- linear factor model

$$(1) \quad \boldsymbol{r}_t = \boldsymbol{\mu}_N + \underbrace{B_N}_{N\times K} \overbrace{\boldsymbol{z}_t}^{K\text{-dim factor vector}} + \boldsymbol{\epsilon}_t$$

  - orthogonal $\boldsymbol{z}_t$ and $\boldsymbol{\epsilon}_t$

$$(2) \quad \boldsymbol{z}_t \sim (0, \overbrace{\Omega}^{\text{f.d.}})$$

$$(3) \quad \boldsymbol{\epsilon}_t \sim (0, \underbrace{\Sigma}_{\text{unbounded dominant e-value}})$$

- vector of pricing errors

$$(4) \quad \breve{\boldsymbol{\alpha}}_N = \boldsymbol{\mu}_N - r_f \mathbf{1}_N - B_N \breve{\boldsymbol{\lambda}}_t$$

- vector of risk premia

$$(5) \quad \breve{\boldsymbol{\lambda}} = (\breve{B}_N^\top \Sigma^{-1} \breve{B}_N)^{-1} \breve{B}_N \Sigma^{-1} (\boldsymbol{\mu}_N - r_f \mathbf{1}_N)$$

- constraint

$$(6) \quad \forall N \; \exists \, \text{finite} \, \delta : \; \breve{\boldsymbol{\alpha}}_N^\top \Sigma^{-1} \breve{\boldsymbol{\alpha}}_N \leqslant \delta$$

Aims (& some Notation) III

Definition (Asymptotic arbitrage)

A sequence of portfolios is said to generate an asymptotic arbitrage opportunity if along some subsequence N': $var(\boldsymbol{r}_t \boldsymbol{w}^a_{N'}) \rightarrow 0$ as $N' \rightarrow \infty$ and $(\boldsymbol{\mu}_{N'} - r_f \mathbf{1}_{N'})^\top \boldsymbol{w}^a_{N'} \geqslant \delta > 0 \; \forall \; N'$.

Why do we care?

- Do you know the factors which expected risk premia load on?

- If not, you will have to deal with **model misspecification**
  - Pricing errors unrelated to factors
    - Managerial skill and analyst recommendation
    - Subjective views
  - Pricing errors related to factors
    - Incorrect means/risk premia or covariances for the factors
    - Missing factors (only some factors are observed)
    - Mismeasured factors (Roll critique).
- How does dealing with model misspecification in the above model (APT) impact portfolio choice?

Challenges I

- Existing theory makes strong and in my view implausible assumptions about covariance matrix for errors. For example, consider a very simple covariance matrix

$$(7) \quad \Sigma_N = \sigma^2 \left[\rho J_N + (1-\rho) I_N\right]$$

  - first $N-1$ e-values are repeated root $1-\rho$ and dominant e-value is $1+\rho(N-1)$
  - dominant e-value clearly unbounded as $N \to \infty$

- paper overcomes this weakness

Challenges I

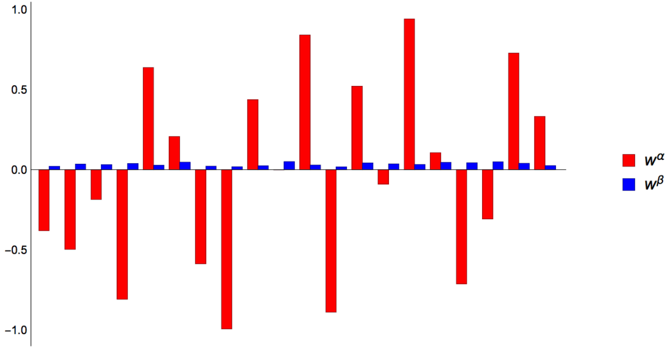
- Existing portfolio choice approach stresses factors and estimation of expected factor risk premia
  - Implementation is hard because expected risk premia are hard to estimate
- this paper tells us to focus on estimating pricing errors instead

Snapshot of Results

- Decompose mean-variance portfolio into alpha and beta portfolios

$$(8) \quad \boldsymbol{w}^{\text{mv}} = \delta^a \boldsymbol{w}^{\alpha} + (1 - \delta^a) \boldsymbol{w}^{\beta}$$

  - α portfolio $\boldsymbol{w}^{\alpha}$ - combination of long and short positions dependent on pricing errors
  - β portfolio $\boldsymbol{w}^{\beta}$ - long portfolio dependent on factor risk premia
- alpha portfolio more important (for large N)
- beta portfolio is a second order issue (for large N) – if we focus on this, it is a bit like colouring a an outline, without really figuring how sensible the outline is.
  - have created a portfolio immune to beta misspecification
  - change focus of empirical portfolio choice from beta estimation to pricing error estimation
  - obtain higher out of sample Sharpe ratios


1.0
0.5
0.0
−0.5
−1.0
w^α
w^β

Can you show that most covariance matrices have unbounded dominant e-values?

- I found it hard to construct examples where the dominant e-values of covariance was bounded.
- Can you prove that 'most' (in a sense to made precise) covariance matrices have an unbounded dominant e-value?

Can you relate your work to the FF portfolios?

- In your decomposition alpha portfolios look like zero cost portfolios and the beta portfolio looks close to the market portfolio
- HML and SMB portfolios are zero cost – made of long and short positions like pricing error portfolios appear to be
- Do positions in the HML and SMB portfolios correspond to the pricing-error part of your portfolio decomposition?

Relationship to robust control

- In robust control, we augment the objective function with a penalty
- The constraint on pricing errors is essentially a penalty – is it related to the Kullback-Leibler divergence?
  - I am pretty sure it is, which means that we can think of a particular pricing error as defining a probability measure. Each probability measure defines a model.
  - But how does the penalty impact portfolio choice?

Dynamic portfolio choice

- From Merton we know that there is an intertemporal hedging demand component in the optimal portfolio when there is a stochastic investment opportunity set
- How is this hedging demand portfolio impacted by pricing errors and how they vary over time?

Equilibrium implications of your portfolios

- If investors hold portfolios with a pricing error component and risk-factor component, what does that imply for asset returns once you impose market clearing?
- Does it mean that asset prices are driven mainly by pricing errors, which could generated by differences in beliefs?
- Should we shift the focus of asset pricing towards differences in beliefs and learning? Maybe this is why some hedge funds don't hire financial economists (Renaissance)

Summary

- Paper with important implications
- Shifts focus of empirical portfolio choice away from beta estimation to pricing error estimation
- Could shift focus of asset pricing towards what drives pricing errors as opposed to betas.
- Need to set up a fund to see how well it works on non-simulated data