Optimal Option Portfolio Strategies by Faias and Santa – Clara

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Why bother with options?

- Options not redundant assets
- Trinomial model: stock and risk - free bond do not span all possible payoffs. Adding a call option would allow complete spanning.
  - Each node in trinomial tree is a dimension
  - Space of all possible payoffs is 3-d
  - Space of payoffs obtainable using stock and bond is 2-d
- Options can help investors obtain a wider range of wealth distributions
- Options should form part of the menu of assets when choosing a portfolio
- But, we generally ignore options in quantitative portfolio choice
- For a hedge fund, a quantitative approach for constructing option strategies would be useful

Why are options ignored?

- Option returns deviate further from normality than stock returns $\rightarrow$ mean – variance analysis less appropriate.
  - Alternative: maximizing power utility over terminal wealth harder technically
- Relatively short time series of option returns – hard to estimate distribution
- Transaction costs more important relative to stocks

Contribution

- Provides a simple method to overcome these difficulties – can be done in XL!
- Tests method out of sample
- Summary

  1. Consider buy and hold allocation. Securities: risk-free asset, ATM calls, ATM puts, 5% OTM calls, 5% OTM puts on S&P500 with 1M to maturity. For each option: long at ask / short at bid: ripoff rule. 8 possibilities using options.
  2. Simulate 1-month S&P500 return via 3 approaches
     - Expanding window
     - Normal distribution with sample moments
     - GEV distribution with sample moments
  3. Allow for stochastic vol by fitting distributions to standardized returns and dividing future return by current estimate of volatility
  4. Maximize simulated expected power utility of terminal wealth by choosing long-only allocation to eight positions and risk-free investment
  5. Out of sample test: find optimal option portfolios one month before maturity – examine return they would have had at maturity

Selected Results

- Including stochastic vol improves performance (get portfolios with Sharpe ratios of 0.6, twice the Sharpe ratio of the underlying index)
- Strategies usually long ATM puts and OTM calls and short OTM puts
  - Jurek & Stafford (2011) find evidence hedge funds sell OTM index puts
- Δ's of strategies are close to zero, Ω's ($\Delta S/V$) are usually negative
  - Optimal option strategies payoff when index does poorly, also highly levered

Comments & Suggestions

- Is the Sharpe ratio a suitable performance measure?
  - Within Black - Scholes framework can obtain Sharpe ratios greater than Sharpe ratio of underlying. Cannot label this as outperformance.

Suggestion 1:

- Choose a suitable benchmark. E.g. choose a portfolio of only risk - free asset and S&P500 index to maximize expected utility.
- Compare performance of optimal options portfolio relative to benchmark by:
  - computing CEQ: $\frac{CEQ^{1-\gamma}}{1-\gamma} = E_t\left[\frac{W_{t+1}^{1-\gamma}}{1-\gamma}\right]$
  - exploiting properties of power utility – see CEQ in terms of moments of log portfolio return:

$$\begin{aligned} CEQ &= e^{\overbrace{\mu_t}^{\text{1st moment}}} \cdot \underbrace{\left(E_t\left[e^{(1-\gamma)[\ln(1+r_{p,t+1})-\mu_t]}\right]\right)^{1/(1-\gamma)}}_{\text{captures impact of 2nd, 3rd, 4th, \ldots moments of log p'folio return on CEQ}} \\ \mu_t &= E_t[\ln(1+r_{p,t+1})] \end{aligned} \tag{1}$$

$$CEQ = W_t e^{\mu_t} \left(E_t \left[e^{(1-\gamma)[\ln(1+r_{p,t+1})-\mu_t]} \right] \right)^{1/(1-\gamma)} \quad (2)$$

- quantifying impact of each moment on CEQ

$$E_t \left[e^{(1-\gamma)[\ln(1+r_{p,t+1})-\mu_t]} \right] = 1 + \sum_{n=1}^{\infty} \frac{(1-\gamma)^n}{n!} E_t[(\ln(1+r_{p,t+1}) - \mu_t)]^n \quad (3)$$

$$= 1 + \frac{(1-\gamma)^2}{2} Var_t[\ln(1+r_{p,t+1})] \quad (4)$$

$$+ \frac{(1-\gamma)^3}{3!} E_t[(\ln(1+r_{p,t+1}) - \mu_t)]^3 \quad (5)$$

$$+ \frac{(1-\gamma)^4}{4!} E_t[(\ln(1+r_{p,t+1}) - \mu_t)]^4 + \ldots \quad (6)$$

- How do these moments change when we have options? Which moments change the most?
- Could this analysis help us unravel the economics underlying the performance of the options strategy?
  - E.g., if $CEQ = W_t^{0.03}$ with options and $CEQ = W_t^{0.01}$ with risk-free bond and index, can we split 3% and 1% up into their constituent components?

Suggestions

Suggestion 2:

- Using Black – Scholes, Heston, or some tractable affine option pricing model (e.g. Heston with jumps) compute Sharpe ratios for the optimal option portfolio. See if you can get 0.6 when the underlying has Sharpe ratio of 0.3. Do optimal portfolios in paper have zero Δ and negative Ω within well known models?

Suggestion 3:

- Find optimal option portfolio when the 5 extremely positive events are removed. How does it differ? Is the Sharpe ratio still 0.6?

Conclusion

- Relatively easy and practical method for constructing option strategies
- Good out of sample performance in terms of Sharpe ratio
- Sharpe ratio may not be best performance metric – try alternatives

Proposition 1

$$\frac{CEQ}{W_t} = e^{\mu_t}\left(1 + \sum_{n=1}^{\infty}(1-\gamma)^n E_t[(\ln(1+r_{p,t+1}) - \mu_t)]^n\right)^{1/(1-\gamma)}, \tag{7}$$

where $m_t = E_t[\ln(1+r_{p,t+1})]$.

Definition 1

The equivalent cts'ly compounded risk-free return, g_t, is defined by

$$e^{g_t} = \frac{CEQ}{W_t}. \tag{8}$$

Proposition 2

The equivalent cts'ly compounded risk-free return, g_t*, can be expressed in terms of central moments of log portfolio returns via*

$$g_t = \mu_t + \frac{1}{1-\gamma}\ln\left(1 + \sum_{n=1}^{\infty}(1-\gamma)^n E_t[(\ln(1+r_{p,t+1}) - \mu_t)]^n\right) \tag{9}$$

or

$$g_t = \mu_t + \frac{(1-\gamma)}{2}Var_t[\ln(1+r_{p,t+1})] + \frac{(1-\gamma)^2}{3!}E_t[(\ln(1+r_{p,t+1}) - m_t)^3] \tag{10}$$

$$+\frac{(1-\gamma)^3}{4!}(E_t[(\ln(1+r_{p,t+1}) - m_t)^4] - 3Var_t[\ln(1+r_{p,t+1})]) + \ldots. \tag{11}$$

Proof of Proposition 1

$$E_t\left[\frac{W_{t+1}^{1-\gamma}}{1-\gamma}\right] = \frac{W_t^{1-\gamma}}{1-\gamma} E_t\left[(1+rp_{t+1})^{1-\gamma}\right]$$

$$= \frac{W_t^{1-\gamma}}{1-\gamma} e^{(1-\gamma)\mu_t} E_t\left[e^{(1-\gamma)[\ln(1+r_{p,t+1})-\mu_t]}\right], \ \mu_t = E_t[\ln(1+r_{p,t+1})]$$

$$CEQ = W_t e^{\mu_t}\left(E_t\left[e^{(1-\gamma)[\ln(1+r_{p,t+1})-\mu_t]}\right]\right)^{1/(1-\gamma)} \quad (12)$$

$$\frac{CEQ}{W_t} = e^{\mu_t}\left(E_t\left[e^{(1-\gamma)[\ln(1+r_{p,t+1})-\mu_t]}\right]\right)^{1/(1-\gamma)} \quad (13)$$

Since $e^x = 1 + \sum_{n=1}^{\infty} \frac{x^n}{n!}$, we have

$$E_t\left[e^{(1-\gamma)[\ln(1+r_{p,t+1})-\mu_t]}\right] = 1 + \sum_{n=1}^{\infty} \frac{(1-\gamma)^n}{n!} E_t[(\ln(1+r_{p,t+1})-\mu_t)]^n \quad (14)$$

$$= 1 + \frac{(1-\gamma)^2}{2} Var_t[\ln(1+r_{p,t+1})] \quad (15)$$

$$+ \frac{(1-\gamma)^3}{3!} E_t[(\ln(1+r_{p,t+1})-\mu_t)]^3 \quad (16)$$

$$+ \frac{(1-\gamma)^4}{4!} E_t[(\ln(1+r_{p,t+1})-\mu_t)]^4 + \ldots \quad (17)$$

Proof of Proposition 2 We can also express the CEQ in terms of a cts'ly compounded growth rate, g_t, which gives us the equivalent cts'ly compounded risk-free return:

$$\frac{CEQ}{W_t} = e^{g_t} \quad (18)$$

$$e^{g_t} = e^{\mu_t}\left(E_t\left[e^{(1-\gamma)[\ln(1+r_{p,t+1})-\mu_t]}\right]\right)^{1/(1-\gamma)} \quad (19)$$

$$g_t = \mu_t + \frac{1}{1-\gamma}\ln\left(E_t\left[e^{(1-\gamma)[\ln(1+r_{p,t+1})-\mu_t]}\right]\right) \quad (20)$$

Hence

$$g_t = \mu_t + \frac{1}{1-\gamma} \ln \left(1 + \sum_{n=1}^{\infty} (1-\gamma)^n E_t[(\ln(1+r_{p,t+1}) - \mu_t)]^n \right). \tag{21}$$

It maybe more useful to start from

$$\frac{CEQ}{W_t} = e^{g_t} = \left(E_t \left[e^{(1-\gamma)\ln(1+r_{p,t+1})} \right] \right)^{1/(1-\gamma)} \tag{22}$$

$$g_t = \frac{1}{1-\gamma} \ln \left(E_t \left[e^{(1-\gamma)\ln(1+r_{p,t+1})} \right] \right) \tag{23}$$

Observe that $\ln \left(E_t \left[e^{(1-\gamma)\ln(1+r_{p,t+1})} \right] \right)$ is a **cumulant generating function** with argument $(1-\gamma)$. It is well known that

$$\ln \left(E_t \left[e^{(1-\gamma)\ln(1+r_{p,t+1})} \right] \right) = \sum_{n=1}^{\infty} (1-\gamma)^n \kappa_{n,t}, \tag{24}$$

where

$$\kappa_{1,t} = \mu_t, \tag{25}$$

$$\kappa_{2,t} = Var_t[\ln(1+r_{p,t+1})] \tag{26}$$

$$\kappa_{3,t} = E_t[(\ln(1+r_{p,t+1}) - m_t)^3] \tag{27}$$

$$\kappa_{4,t} = E_t[(\ln(1+r_{p,t+1}) - m_t)^4] - 3Var_t[\ln(1+r_{p,t+1})]. \tag{28}$$

[Can also find tractable expressions for $\kappa_{5,t}$ and $\kappa_{6,t}$] Hence

$$g_t = \mu_t + \frac{(1-\gamma)}{2} Var_t[\ln(1+r_{p,t+1})] + \frac{(1-\gamma)^2}{3!} E_t[(\ln(1+r_{p,t+1}) - m_t)^3] \tag{29}$$

$$+ \frac{(1-\gamma)^3}{4!} (E_t[(\ln(1+r_{p,t+1}) - m_t)^4] - 3Var_t[\ln(1+r_{p,t+1})]) + \ldots \tag{30}$$