

Speculative Betas

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Overview I

- Investing in assets with low CAPM betas seems to be much better than investing in assets with high CAPM betas. [Black (1972) and Black, Jensen & Scholes (1972)]
 - 1 USD invested in a portfolio of the lowest decile of beta stocks on January of 1968 would have yielded 11 USD in real terms at the end of December 2008. 1 USD invested in the highest decile of beta stocks would have yielded around 0.64 USD. [Baker, Bradley, and Wurgler (2011)]
- This paper offers a potential resolution via a simple theoretical model.
- The model makes other empirical predictions which seem to be true.

Overview II

A model (multi-asset extension of Chen, Hong & Stein (2002) with some simplifications) of

- mutual funds who disagree about the mean size of aggregate shocks to dividends and cannot sell short
- hedge funds who have correct beliefs and are unconstrained

which implies that

- a security market line with a negative slope for high beta assets when disagreement is high (resolves puzzle)
- assets with higher betas are shorted more heavily and have higher turnover, which rises with disagreement (additional prediction)

Assumptions & Intuition

- Assumptions about who cannot short sell and who is unconstrained are sensible.
- Plenty of circumstantial evidence on disagreement.
- If a fraction of MF's are relatively pessimistic they will want to short some stocks. They cannot. This leads to overpricing and a reduction in expected risk premia. If disagreement is large enough, reduction can be large enough to make risk premia decreasing wrt beta.

Benchmark Static Model

- 1 period, 2 dates: 0,1
- N dividends

$$\tilde{d}_i = b_i \cdot \underbrace{\underbrace{\tilde{z}}_{\sim N(\bar{z}, \sigma_z^2)} + \underbrace{\tilde{\epsilon}_i}_{\sim N(0, \sigma_\epsilon^2)}}_{\text{Cov}(\tilde{z}, \tilde{\epsilon}_i) = 0}$$

- i 'th risky asset pays \tilde{d}_i at date 1
- exogenous risk-free rate
- Continuum of agents have identical mean-variance preferences

$$E[W^k] - \underbrace{\frac{1}{\gamma}}_{\gamma: \text{risk tolerance}} \cdot \text{Var}[W_k]$$

- Price of i 'th risky asset

$$P_i = \frac{b_i \bar{z} - \frac{1}{\gamma} (\sigma_z^2 + \frac{1}{N} \sigma_\epsilon^2)}{1 + r} = \frac{\overbrace{E^Q[\tilde{z}_i]}^{\text{risk-neutral exp. of dividend}}}{1 + r}$$

- Expected excess return on i 'th risky asset

$$E[\tilde{R}_i] = \frac{1}{\gamma} \left(\sigma_z^2 + \frac{1}{N} \sigma_\epsilon^2 \right)$$

Static Model: Mutual Funds & Hedge Funds

- Distinguish between **Mutual Funds** (MF's) and **Hedge Funds** (HF's)
- Fraction α of agents are MF's, who cannot sell short
- Fraction $1 - \alpha$ of agents are HF's, unconstrained

- Type A MF's have optimistic beliefs

$$E^A[\tilde{z}] = \bar{z} + \lambda$$

- Type B MF's have pessimistic beliefs

$$E^B[\tilde{z}] = \bar{z} - \lambda$$

- For b_i sufficiently large, price of i 'th risky asset

$$P_i = \frac{b_i \bar{z} - \frac{1}{\gamma} (\sigma_z^2 + \frac{1}{N} \sigma_\epsilon^2) + \pi_i}{1 + r} = \frac{E^{Q^k}[\tilde{z}_i]}{1 + r}$$

- Expected excess return on i 'th risky asset

$$E[\tilde{R}_i] = \frac{1}{\gamma} \left(\sigma_z^2 + \frac{1}{N} \sigma_\epsilon^2 \right) - \pi_i$$

- Type B MF's would like to short assets which load heavily on aggregate shock \tilde{z} (high b assets). They cannot, so prices of high b assets are high relative to benchmark model: defined as **overpricing**.

- Increase fraction of MF's: more constrained agents and π_i larger
- Increase disagreement: π_i larger

Rewrite results in CAPM form

- Expected excess return on i 'th risky asset (when short sales constraint binds)
- Benchmark static model

$$E[\tilde{R}_i] = \underbrace{\frac{\text{Cov}(\tilde{R}_i, \tilde{R}_M)}{\text{Var}(\tilde{R}_M)}}_{\beta_i} \cdot \frac{1}{\gamma} \left(\sigma_z^2 + \frac{1}{N} \sigma_\epsilon^2 \right)$$

- Static model with MF's and HF's

$$E[\tilde{R}_i] = \beta_i \frac{1}{\gamma} \left(\sigma_z^2 + \frac{1}{N} \sigma_\epsilon^2 \right) (1 - \theta \kappa(\lambda)) + \frac{1}{\gamma N} \sigma_\epsilon^2 \theta (1 + \kappa(\lambda))$$

- Important term is the factor $1 - \theta \kappa(\lambda)$
- When disagreement is large, $\kappa(\lambda)$ is large: can have $1 - \theta \kappa(\lambda) < 0$
- Expected excess return can be **decreasing** wrt asset beta!

Dynamic Extension with MF's & HF's

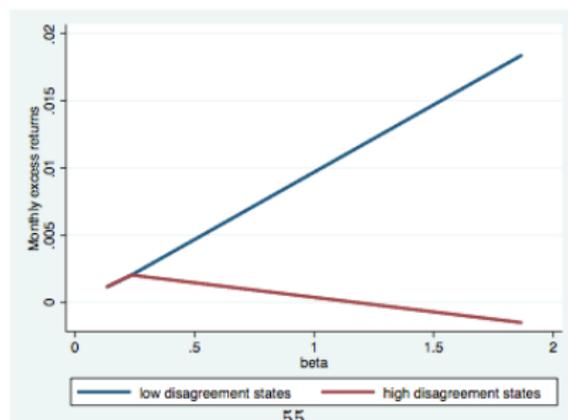
- $t \in \mathbb{N}_0$
- Standard OLG extension
- Stochastic disagreement, $\tilde{\lambda}_t \in \{0, \lambda\}$, where disagreement changes with prob. $1 - \rho$ and persists with prob. ρ .
- As before, high b assets are overpriced.
- What's new?
 - $\tilde{\lambda}_t$ is a new risk factor. Date- t prices will depend on realization of disagreement. Given that agents are forced to sell assets after a period, the covariance of risky assets with the new risk factor will be priced, creating new term in the expected excess return for assets which Type B MF 's want to sell short (high b assets): ω_j

$$\begin{aligned} \overbrace{E[\tilde{R}_i(0)]}^{\text{zero disagreement}} &= \frac{1}{\gamma} \left(\sigma_z^2 + \frac{1}{N} \sigma_\epsilon^2 \right) + \omega_j \\ \underbrace{E[\tilde{R}_i(\lambda)]}_{\text{non-zero disagreement}} &= \frac{1}{\gamma} \left(\sigma_z^2 + \frac{1}{N} \sigma_\epsilon^2 \right) + \omega_j - f\pi_j \end{aligned}$$

When disagreement is highly persistent, risk associated with possible changes in disagreement is small: ω_j is negligible.

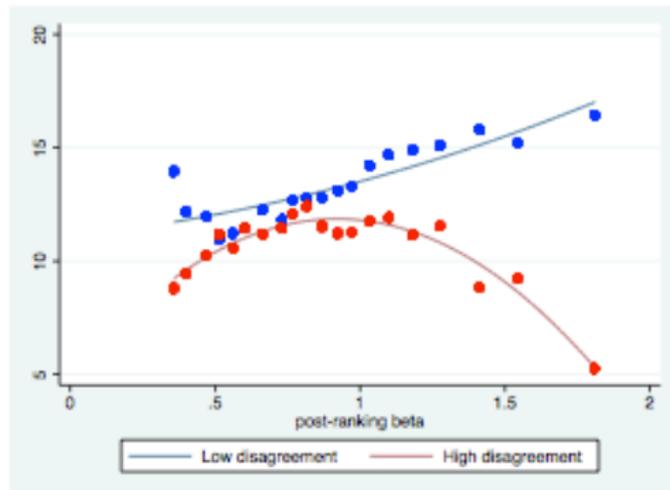
Dynamic Extension: Implications for Security Market Line & Turnover

- For high b assets:
 - $E[\tilde{R}_i(0)]$ increasing with β_i
 - $E[\tilde{R}_i(\lambda)]$ **decreasing** with β_i when λ sufficiently large
 - When $\lambda_t = \lambda > 0$, turnover is higher



(c) $\lambda = .015$

A nice plot from the data



(d) 12 months equal-weighted return

Explore properties of set of SDF's

- In the static model we have

$$P_i = \frac{b_i \bar{z} - \frac{1}{\gamma} \left(\sigma_z^2 + \frac{1}{N} \sigma_\epsilon^2 \right) + \pi_i}{1 + r} = \frac{E^{\mathbb{Q}^k}[\tilde{z}_i]}{1 + r}$$

- Markets are incomplete: instead of a unique risk-neutral measure have a set of risk-neutral measures, $\{\mathbb{Q}^k\}_k$
- Derive the family of risk-neutral probability measures or equivalently SDF's which price assets. Discuss their properties: market price of risk. Hopefully more plausible than properties of SDF's from alternative models.

Compare with Frazzini & Pedersen (2010)

- Frazzini & Pederson (2010): **leverage constraints** lead to high beta assets having low alpha.
- What does their model imply for turnover of high beta assets?
- How does the set of SDF's differ from the set of possible SDF's in Frazzini & Pederson (2010). Is your SDF set more plausible?

Dynamic models with wealth effects can also flatten security market line

- Mention this literature.
- In a consumption-based model, relationship between risk and return can be negative depending on assumptions about **preferences** and **probability distributions** for consumption growth, e.g. Backus & Gregory (1993). Can this be extended to firm-level?
- Flattening of security market line can come from
 - Roll Critique
 - Hedging Demands
 - Differences in Beliefs
 - Short sales constrains
- Why is your approach better than dynamic models with the above features?

Dynamic Models

- **Roll Critique** related papers: beta wrt aggregate stock market is not the same as beta wrt aggregate wealth. Can model in Pollet & Wilson (2008) be extended to explain this?
 - Changes in variance of the stock market only weakly related to changes in aggregate risk and subsequent stock market excess returns. Is same true for betas?
- **Hedging demand** takes us away from standard CAPM. Guo & Whitelaw (2006) find that expected returns are driven mainly by hedging demand. This can explain why aggregate stock market vol and conditional risk premium are weakly related. This might explain why individual stocks with low betas have high expected risk premia. See also Scruggs (1998)

Dynamic Models II

- **Differences in Beliefs:** May not need short sales constraints when you have wealth effects. See Bhamra & Uppal (2011): easy to extend to multiple assets.
- **Differences in Beliefs and Short Sales Constraints:**
 - models with one stock: Gallmeyer & Hollifield (2008) and Chabakauri (2011).
 - models with multiple stocks: Hugonnier (2012) (log utility, differences in beliefs, constraints, 2 agents, multiple assets). Using his mathematical results, look at the security market line. What are new insights your approach?

Roll Critique

- No arbitrage $\Rightarrow \exists \Lambda > 0$ s.t.

$$E_t[dR_{i,t} - r_t dt] = E_t \left[dR_{i,t} \cdot \frac{d\Lambda_t}{\Lambda_t} \right]$$

- Assume \exists rep. inv. with value function

$$J(W_t) = \frac{(aW_t)^{1-\gamma}}{1-\gamma}, \text{ a constant}$$

$$\frac{d\Lambda_t}{\Lambda_t} = -\gamma \frac{dW_t}{W_t} + \text{drift terms}$$

- W_t is human capital H_t plus financial wealth F_t . Share of human capital is $s_t = H_t/W_t$

$$\frac{dW_t}{W_t} = s_t \frac{dH_t}{H_t} + (1-s_t) \frac{dF_t}{F_t} + \text{drift terms}$$

$$E_t[dR_{i,t} - r_t dt] = \gamma s_t E_t \left[dR_{i,t} \cdot \frac{dH_t}{H_t} \right] + \gamma(1-s_t) E_t \left[dR_{i,t} \cdot \frac{dF_t}{F_t} \right]$$

$$= \gamma s_t \beta_{i,H,t} \sigma_{H,t}^2 + \gamma(1-s_t) \cdot \underbrace{\beta_{i,F,t}}_{\text{CAPM beta}} \cdot \sigma_{F,t}^2$$

- Van Nieuwerburgh, Lustig & Verdelhan (2012): estimate $s_{H,t} = 0.9$.
- Gomme & Rupert (2004): Labor's share in income rises in recessions
- Weakens relationship between CAPM beta and expected risk premia.
- Hard getting negative relationship hard? Can $\beta_{i,H,t}$ and $\beta_{i,F,t}$ be negatively related?

Hedging Demand

- No arbitrage + assume \exists rep. inv. with value function

$$J(W_t) = \frac{(a_t W_t)^{1-\gamma}}{1-\gamma}, \quad a_t \text{ stochastic}$$

$$E_t[dR_{i,t} - r_t dt] = (\gamma - 1)\beta_{i,a,t}\sigma_{a,t}^2 + \gamma s_t \beta_{i,H,t}\sigma_{H,t}^2 + \gamma(1 - s_t) \cdot \underbrace{\beta_{i,F,t}}_{\text{CAPM beta}} \cdot \sigma_{F,t}^2$$

- Further weakens relationship between CAPM beta and expected risk premia.