Robust Assessment of Hedge Fund Performance through Nonparametric Discounting by Almeida and Garcia

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Overview I

- Do hedge funds really provide risk – adjusted returns much higher than the risk - free rate?
- Or is the risk – adjusted inaccurate
- Will a better risk adjustment lead to lower risk – adjusted returns?
- How can we carry out the risk adjustment more accurately?

Overview II

Theoretical Asset Pricing

Definition 1

m is a SDF if

$$\forall \text{ assets } i, \; E[mR_i] = 1. \tag{1}$$

Empirical Asset Pricing

- Using some empirical methodology [e.g. Hansen & Jagannathan, (1991)], find that

$$m = a + \sum_{j=1}^{J} b_j \underbrace{f_j}_{\text{factor, usually a return of a basis asset}} \tag{2}$$

- Using empirically determined SDF, price hedge fund return

$$E[mR_{HF}] = \alpha \tag{3}$$

- α large and positive
- hedge fund offers high risk adjusted return or the SDF has been determined inaccurately

This paper

- New methodology for determining a family of SDF's from time series data on returns
- SDF can be a non linear function of factors (returns on basis assets)
  - presence of non linearities in hedge fund returns: Fung & Hsieh (2001), Mitchell & Pulvino (2001), Agarwal & Naik (2004)
  - usual approach: include options in set of basis assets, keep *m* linear
- relative to linear SDF, non linear SDF's give lower α
  - some of the non linear SDF's capture better higher moments – reduction in α
  - accounting for extreme events leads to a reduction in α
- determine SDF via Fenchel dual problem – easier than primal approach
  - dual problem is a portfolio choice problem

Estimating a SDF I

- for all assets, i

$$E[mR_i] = 1 \tag{4}$$

  - this is an average over states of the economy $\sum_{k=1}^{K} p(\omega_k)m(\omega_k)R_i(\omega_k) = 1$

- if Ergodic Hypothesis is true (replace average over states with time series average – can use time series data), then

$$\underbrace{\lim_{T\to\infty} \sum_{t=1}^{T} \frac{m_t R_{i,t}}{T}}_{\text{time series average}} = \underbrace{E[mR_i]}_{\text{average over states}} \tag{5}$$

- For large T

$$\sum_{t=1}^{T} \frac{m_t R_{i,t}}{T} = 1 \tag{6}$$

$$\frac{1}{T} \sum_{t=1}^{T} m_t (R_{i,t} - a^{-1}) = 0 \tag{7}$$

where $a = \sum_{t=1}^{T} \frac{m_t}{T}$

- Need an extra condition to determine SDF

Estimating a SDF II

- Hansen – Jagannathan (1991)

$$\hat{m}_{HJ} = \frac{1}{2} argmin_{\{m_t\}_{t=1}^{T}} \overbrace{\frac{1}{T}\sum_{t=1}^{T}[(m_t)^2 - a^2]}^{\text{variance of SDF}} \tag{8}$$

s.t.

$$\forall i \in \{1,\ldots,I\},\ \frac{1}{T}\sum_{t=1}^{T} m_t(R_{i,t} - a^{-1}) = 0 \tag{9}$$

$$\sum_{t=1}^{T} \frac{m_t}{T} = a \tag{10}$$

$$\forall t \in \{1,\ldots,T\},\ m_t > 0. \tag{11}$$

  - Alternatives to minimizing variance: Snow (1991), Stutzer (1995), Bansal & Lehmann (1997), Bernardo & Ledoit (2000), Cerny (2003)
- This paper: replace (8) by $\hat{m}_{MD} = \frac{1}{2} argmin_{\{m_t\}_{t=1}^{T}} \frac{1}{T}\sum_{t=1}^{T}\phi(m_t)$, where $\phi(\cdot)$ is a convex function of the form

$$\phi^{\gamma}(m) = \frac{m^{1+\gamma} - a^{1+\gamma}}{\gamma(1+\gamma)} \tag{12}$$

Fenchel dual problem

- Instead of finding $\{m_t\}_{t=1}^T$ find $\{\lambda_i\}_{i=1}^I$: $I < T$ (easier)

$$\hat{\lambda} = argsup_{\epsilon \in \mathbb{R}, \lambda} a\epsilon - \frac{1}{T}\sum_{t=1}^{T} \psi^{\gamma}(\epsilon + \lambda^T (R_t - \frac{1}{a}1_I)) \quad (13)$$

$$\psi^{\gamma}(z) = \underbrace{\sup_{w>0} zw - \phi^{\gamma}(w)}_{\text{Fenchel dual}} \quad (14)$$

$$\hat{m}_{t,MD} = a\frac{\left(a^{\gamma} + \gamma\hat{\lambda}^T (R_t - \frac{1}{a}1_I)\right)^{1/\gamma}}{\frac{1}{T}\sum_{t=1}^{T}\left(a^{\gamma} + \gamma\hat{\lambda}^T (R_t - \frac{1}{a}1_I)\right)^{1/\gamma}} \quad (15)$$

- SDF is non linear
  - Prior literature: SDF linear but with returns from assets with non linear payoffs
- $\gamma = 1$: reduces to linear SDF. Hansen – Jagannathan (1991) is a special case of this paper.
- Dual problem can be interpreted as a portfolio choice problem
  - investor with HARA utility: $-\gamma$ cautiousness parameter
  - $\{\lambda_i\}_{i=1}^I$ are units of wealth invested in each risky asset

What is going on?

- Researcher chooses a set of basis assets: $\{R_i\}_{i=1}^{I}$
- HARA investor with cautiousness, $-\gamma$ chooses portfolio of the basis assets, i.e. λ
- λ plus the basis assets gives us the nonlinear SDF, m
- for each γ get a different SDF: a family of SDF's

Family of SDF's

Portfolios (λ)

- More cautious investors ($-\gamma > 0$ [see Eq (14) in paper]) and larger: put more weight on skewness and kurtosis
- $-\gamma > 0$ and larger: more weight on skewness and kurtosis in SDF
- returns: S&P 500 (equity), RU2000 (equity, size – spread), MSCI (emerging market index), 10yrTr, BAA, r_f
  - all investors: short BAA, long all others
  - more cautious investors: more weight on r_f rises, less weight on risky assets

SDF volatility $\equiv$ market price of risk

- options added to set of basis assets: higher market price of risk and more sensitivity to extreme events
- More cautious HARA investor: higher market price of risk
- higher market price of risk: will lower performance of any risky strategy correlated with SDF
- extreme events v. important for correct analysis of hedge fund performance if HF's sell puts!

Pricing errors

- Lowest for $-\gamma = 3, 2, 0$
- Large for $-\gamma = 1$ (HJ)

Hedge fund performance

- increasing cautiousness of HARA investor: SDF more volatile, lower pricing errors
- adding options to basis assets: SDF higher during very bad states of the world and SDF more volatile
- increasing cautiousness of HARA investor and adding options to basis assets lowers hedge fund performance

Suggestions: Fenchel duality

- Fenchel duality appears in many places: physics (with v clear interpretations), finance (with less clear interpretations)
  - Consumption – portfolio choice problems with constraints: more easily solved using Fenchel dual [Cvitanic & Karatzas (1992)]
  - Social planner's optimization problem where individual agents have recursive preferences: more easily solved using Fenchel dual [Dumas, Uppal & Wang (2000)]
    - Fenchel dual can be interpreted as problem of a robust investor
- This paper: why does the Fenchel dual problem in this paper appear in the form of a portfolio choice problem
- Can we find a unified economic interpretation for Fenchel dual problems in general?

Suggestions: Testing the methodology

- Compare your methodology for estimating SDF with others in a framework where we actually know what the SDF is
  - consumption – based asset pricing model, complete markets
    - e.g., N dividend trees subject to rare disasters, single agent with power utility or several agents with catching up with Jones preferences (different curvature parameters), complete markets
    - compute unique SDF, m, directly
    - use simulated time series data on returns to estimate SDF, $\hat{m}$: measure $||\hat{m} - m||$
  - consumption – based asset pricing model, incomplete markets
    - each agent's SDF can be computed directly, m_h
    - use simulated time series data on returns to estimate SDF, $\hat{m}$: which SDF is it closest to?