Discussion: Equilibrium Wealth Share Dynamics by Ravi Bansal, Colin Ward, & Amir Yaron

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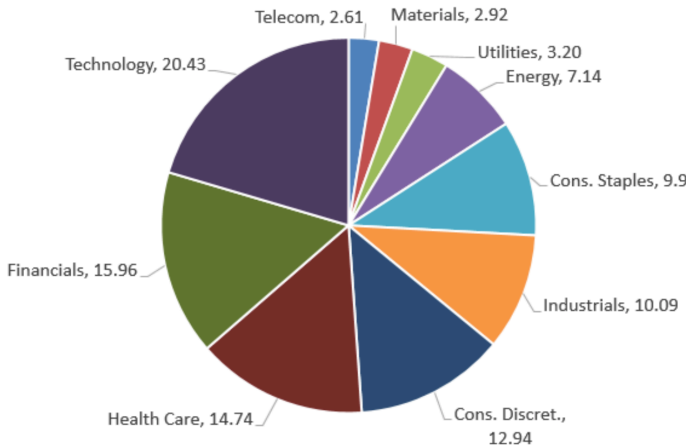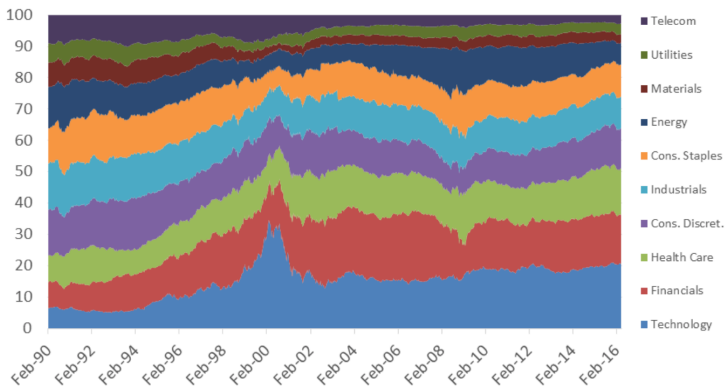
2017

Composition of Economy I

- Economy is divided into different sectors
- The relative wealth of each sector is different and varies over time
- Why do sectors vary in size, expected growth rate and riskiness?
    - For example, why has the Tech Sector grown so fast recently and how risky is it?
- How do above characteristics impact expected risk premia?
- Welfare implications of sectoral composition
    - Is it good or bad to have one dominant sector?
    - What are the pros and cons for the UK of having a relatively large financial sector?
- Which new sectors will have arisen 50 years from now?

S&P 500 Current Sector Weightings (%)
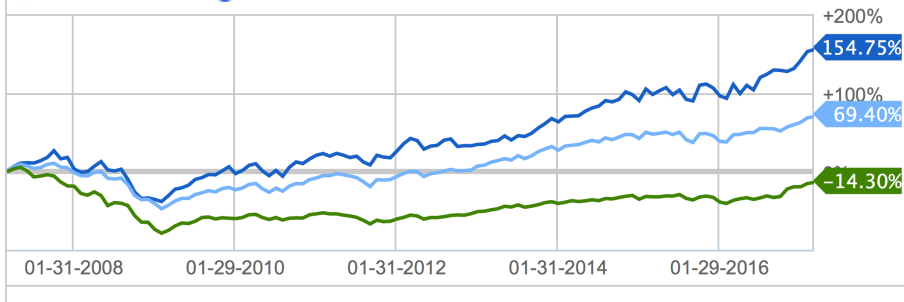
Telecom, 2.61
Materials, 2.92
Utilities, 3.20
Energy, 7.14
Technology, 20.43
Cons. Staples, 9.9
Industrials, 10.09
Financials, 15.96
Cons. Discret., 12.94
Health Care, 14.74

S&P 500 Historical Sector Weightings: 1990-Present (% Scale)

01-31-2017 ■ Information Technology 140.56% ■ S&P 500 INDEX 61.99% ⊗
■ Financials -19.78% ⊗

Can we do this exciting work?

- Intuition captured in existing models: when Apple was a small firm it did not contribute much to systematic risk so it's expected risk premium was small back then (e.g. Pastor & Veronesi (2009))
- Data: this is wrong – small sectors have large expected risk premia
- Implication: we are not well equipped to understand asset pricing at the sector level
- This paper: helps us get started

This paper

- Measure sector size by wealth share
- Why do smaller sectors have larger expected risk premia, even though their contribution to systematic risk is small?
- Why does an increase in sector size raise Tobin's q?

Seek answers to above in a 2 sector production economy with imperfect substitutability across goods and exogenous demand shocks

Figure 1: Risk premia, Tobin's q, and wealth shares
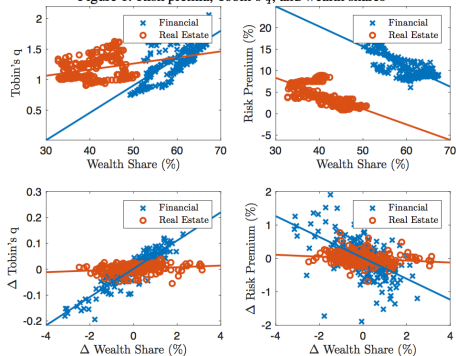
Data are quarterly from 1952Q3 until 2015Q4. Tobin's Q divides the household's invested wealth in the sector by the sector's current-cost capital stock. Wealth shares are market capitalization shares. Risk premia are estimated by the fitted values of a predictive regression of annual excess returns on today's cash flow yield. The top two panels report the data in levels. The bottom two panels report the data in first differences. Regression lines are color-coded and overlay the data.

This paper's assertions

- Exogenous demand shocks combined with imperfect substitutability of of goods creates creates large desire to hedge against exogenous demand shocks
- Large hedging demand drives up expected risk premia for smaller sectors and leads to Tobin's q increasing with sector size

Model Summary I

Assertions rest on exploration of a 2 sector production economy, single EZW representative agent with CES aggregator that has exogenous stochastic weights

- 2 sectors (one for each good)

$$dK_{n,t} = \phi_n\left(\frac{I_{n,t}}{K_{n,t}}\right)K_{n,t}dt + \sigma_n K_{n,t}dZ_{n,t} \tag{1}$$

  - quadratic adjustment costs

$$\phi_n(x) = x - \frac{1}{2}\kappa_n x^2 \tag{2}$$

- $E_t[dZ_{1,t}dZ_{2,t}] = \varphi dt$

Model Summary II

- CES aggregator

$$C_t = \left(\Omega_t^{\frac{1}{\epsilon}} D_{1,t}^{1-\frac{1}{\epsilon}} + (1-\Omega_t)^{1-\frac{1}{\epsilon}} D_{2,t}^{\frac{1}{\epsilon}}\right)^{\frac{1}{1-\frac{1}{\epsilon}}} \tag{3}$$

- $\eta = 1 - 1/\epsilon$, degree of complementarity ($\eta = 0$ perfect sub's, $\eta \to \infty$ perfect comp's)
- $\Omega_t \in (0,1)$ is a **relative demand shock**
  - High Ω increases demand for good 1, decreases demand for good 2

$$\frac{D_{1,t}}{C_t} = \left(\frac{p_{1,t}}{p_t}\right)^{-\epsilon} \Omega_t, \quad \frac{D_{2,t}}{C_t} = \left(\frac{p_{2,t}}{p_t}\right)^{-\epsilon} (1-\Omega_t) \tag{4}$$

- $\Omega_t \in \{\Omega_1, \ldots, \Omega_M\}$, transitions governed by exogenously specified Markov chain
  - discrete state space for Ω used to make numerical solution easier

$$J_t = \frac{(K_t \overbrace{v(k_t, \Omega_t)}^{\text{ode system}})^{1-\gamma}}{1-\gamma} \tag{5}$$

  - I shall think of Ω as a **mean-reverting** and continuous process on $(0,1)$
- dynamically complete markets

Simplify intuition for results on expected risk premia and sector size

Demand shocks have a larger impact on utility when supply is scarce

- use EZW agent to ensure above shocks to utility translate into shocks to SDF

SDF – EZW preferences and demand shocks I

- Demand shocks are priced because agent's consumption is sensitive to demand and she is not indifferent towards timing of intertemporal risk (latter is what matters)

  - Unexpected component of SDF, Λ (used for pricing risk)

$$d \ln \Lambda_t - E_t[d \ln \Lambda_t] = -\gamma \overbrace{(d \ln K_t - E_t[d \ln K_t])}^{\text{aggregate shock}} \tag{6}$$

$$- \left[\frac{1}{\psi} \frac{\partial \ln c(k_t, \Omega_t)}{\partial k_t} + \left(\gamma - \frac{1}{\psi}\right) \frac{\partial \ln v(k_t, \Omega_t)}{\partial k_t}\right] \overbrace{(dk_t - E_t[dk_t])}^{\text{shock to first sector}} \tag{7}$$

$$- \left[\frac{1}{\psi} \frac{\partial \ln c(k_t, \Omega_t)}{\partial \Omega_t} + \left(\gamma - \frac{1}{\psi}\right) \frac{\partial \ln v(k_t, \Omega_t)}{\partial \Omega_t}\right] \underbrace{(d\Omega_t - E_t[d\Omega_t])}_{\text{relative demand shock}} \tag{8}$$

- $K_t = (K_{1,t}^{\eta} + K_{2,t}^{\eta})^{1/\eta}$, $k_t = (K_{1,t}/K_t)^{\eta}$
- $c_t = C_t/K_t$, v_t – dependence of utility on k_t and Ω_t

SDF – EZW preferences and demand shocks II

$$d \ln \Lambda_t - E_t \left[d \ln \Lambda_t \right] = \text{irrelevant stuff} + \left(\gamma - \frac{1}{\psi} \right) \frac{\partial \ln v(k_t, \Omega_t)}{\partial \Omega_t} \underbrace{(d\Omega_t - E_t[d\Omega_t])}_{\text{relative demand shock}} \quad (9)$$

- $\left(\gamma - \frac{1}{\psi} \right) \frac{\partial \ln v(k_t, \Omega_t)}{\partial \Omega_t} \sqrt{Var_t[d\Omega_t]}/dt$ is the price of risk linked to demand shocks for good 1

- $\frac{\partial \ln v(k_t, \Omega_t)}{\partial \Omega_t}$ larger when k is small – **you need to plot this**

- $\gamma - \frac{1}{\psi}$ more positive when agent has a stronger preference for earlier resolution of intertemporal risk

1. Demand shocks for good 1 have a larger impact on utility when supply of good 1 is scarce and good 2 is not a great substitute

2. Preference for early resolution of intertemporal risk implies shock to utility translates into shock to SDF

3. Obtain increase in price of risk for demand shocks to good 1 when k is small

4. Will drive up expected risk premium for sector 1 when it is small, provided demand shocks for good 1 impact unexpected returns on sector 1

Improve connection to hedging demand I

- Improve translation of discount rate variation story into hedging demand story
- How is $d \ln \Lambda_t - E_t \left[d \ln \Lambda_t \right] = \text{irrelevant stuff} + \left(\gamma - \frac{1}{\psi} \right) \frac{\partial \ln v(k_t, \Omega_t)}{\partial \Omega_t} \underbrace{(d\Omega_t - E_t[d\Omega_t])}_{\text{relative demand shock}}$

  linked to hedging demand?

- Obtain portfolio weight vector $\underline{\phi}_t = (\phi_{1,t}, \phi_{2,t})^\top$ from consumption-portfolio choice problem of agent (Merton, previous century)

$$\underline{\phi}_t = \overbrace{\frac{1}{-W^2 J_{WW}/J_W} (\Sigma_t^\top \Sigma_t)^{-1}(\underline{\mu}_t - r_t \underline{1})}^{\text{mean-variance demand}} \tag{10}$$

$$+ \overbrace{\left(-1/W^2 J_{WW} \right) (\Sigma_t^\top \Sigma_t)^{-1} (\Sigma_{x,t}^\top \Sigma_t) J_{W\underline{x}_t}}^{\text{hedging demand}} \tag{11}$$

- $\underline{x}_t = (k_t, \Omega_t)^\top$
- $d\underline{x}_t - E_t[d\underline{x}_t] = \Sigma_{x,t} d\underline{B}_t$
- $d\underline{R}_t - E_t[d\underline{R}_t] = \Sigma_t d\underline{B}_t$

Improve connection to hedging demand II

- From SDF, we know that

$$\partial J / \partial W = K^{-\gamma} c(\underline{x}_t)^{-1/\psi} v(\underline{x}_t)^{-(\gamma - 1/\psi)} \tag{12}$$

- Obtain

$$\underline{\phi}_t = \overbrace{\frac{1}{\gamma} (\Sigma_t^\top \Sigma_t)^{-1} (\underline{\mu}_t - r_t \underline{1})}^{\text{mean-variance demand}} \tag{13}$$

$$\overbrace{+ (1 - 1/\gamma) \left(/ (\partial \ln K_t / \partial \ln W_t) \right) (\Sigma_t^\top \Sigma_t)^{-1} (\Sigma_{x,t}^\top \Sigma_t) \left(\frac{1}{\psi} \partial_{\underline{x}_t} \ln c + \left(\gamma - \frac{1}{\psi} \right) \partial_{\underline{x}_t} \ln v \right)}^{\text{hedging demand}} \tag{14}$$

- Relevant portion of hedging demand depends on

$$Cov_t[d\Omega_t, dR_{i,t}] \left(\gamma - \frac{1}{\psi} \right) \frac{\partial \ln v_t}{\partial \Omega_t} \tag{15}$$

- This is precisely what drives discount rate variation leading to higher expected risk premium for smaller sector
- Need EZW for hedging demand to appear in shocks to SDF

Improve connection to hedging demand III

Covariance of shocks to SDF from demand shocks with returns $\equiv$ hedging demand against demand shocks

Look at case with no adjustment costs

- Risk premia effects not driven by adjustment costs
- Model solution will be much simpler without them

Intuition for Tobin's q

- Tobin's q smaller when sector share decreases, because of less consumption of output from smaller sector
- Is it that simple? Do you need Epstein-Zin for this?
- Would be nice to have stronger connection between economics and the mathematical model
- This would make connection between hedging demand and Tobin's q explicit
- Perhaps a perturbation expansion around case of $\kappa_n = 0$ (I know you use $\kappa_n = 10$, but it's a start)

Application: Financial and Real Estate Wealth

- Does it really make sense to understand relative fluctuations in financial and real estate wealth in a model with no household leverage?

Origins of Demand Shocks

- Problem: explaining what we did not/don't understand via exogenous shocks
- Where do demand shocks come from?
- We don't know the future range of goods available to us. Smart innovators understand and anticipate needs of humans and can figure out how to meet them via creating new products. The creation of new products spurs a process of two-sided learning, where consumers learn about what is available and learn how to use it to meet their needs, while innovators try and improve their understanding of human needs.
- Can we model this?

Conclusions

- Simplify intuition for how demand shocks impact SDF when supply is scarce
- Explicitly connect discount rate shocks driven by demand shocks to hedging against demand shocks
- Consider no adjustment cost case for clean expressions for $v(k_t, \Omega_t)$
- Equations for Tobin's q showing economics
- Why can you ignore household leverage?

Expected risk premia and sector size I

Theorem 1

Suppose returns are continuous. If the CAPM holds and all sectors have equal return volatilities and the correlation of returns across different sectors is the same (symmetry assumptions), then expected risk premia are increasing in sector size

Proof

- Dynamic intertemporal asset pricing equation

$$E_t[dR_{i,t} - r_t dt] = -E_t\left[\frac{d\Lambda_t}{\Lambda_t} dR_{i,t}\right] \tag{16}$$

$$\tag{17}$$

Expected risk premia and sector size II

- CAPM SDF

$$\frac{d\Lambda_t}{\Lambda_t} = -r_t dt - (dR_{m,t} - E_t[dR_{m,t}]) \tag{18}$$

$$\underbrace{dR_{m,t}}_{\text{market return}} = \sum_{i=1}^{I} \underbrace{S_{i,t}}_{\text{relative wealth of sector } i} dR_{i,t} \tag{19}$$

- Returns

$$dR_{i,t} = \mu_{i,t} dt + \sigma_{i,t} dZ_{i,t} \tag{20}$$

$$E_t[dZ_{i,t} dZ_{j,t}] = \rho_{ij,t} dt \tag{21}$$

Expected risk premia and sector size III

- Exploiting CAPM assumption

$$\mu_{i,t} - r_t = \sigma_{i,t}^2 S_{i,t} + \sigma_{i,t} \sum_{j \neq i} S_{j,t} \rho_{ij,t} \sigma_{j,t} \tag{22}$$

$$\tag{23}$$

- Exploiting symmetry assumptions

$$\mu_{i,t} - r_t = \sigma_{i,t}^2 S_{i,t} + \sigma_{i,t} \sum_{j \neq i} S_{j,t} \rho_{ij,t} \sigma_{j,t} \tag{24}$$

$$= \sigma_t^2 S_{i,t} + \rho_t \sigma_t^2 \sum_{j \neq i} S_{j,t} \tag{25}$$

$$= \sigma_t^2 S_{i,t} + \rho_t \sigma_t^2 (1 - S_{i,t}) \tag{26}$$

- $\mu_{i,t} - r_t$ increasing with $S_{i,t}$ if $1 > \rho_t$
∎