

**Discussion: Generalized Disappointment
Aversion and Asset Prices**
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1 Motivation

- ▶ Equity premium puzzle – Mehra and Prescott (1985)

1.1 State-dependent and countercyclical risk aversion

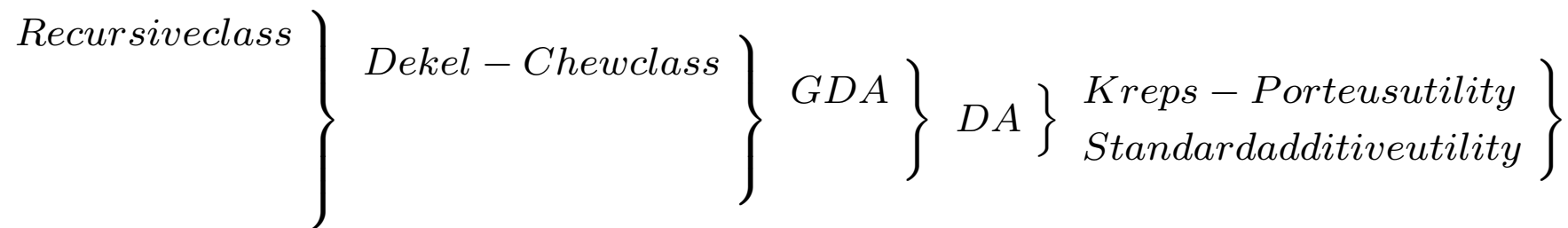
- ▶ For a representative agent economy to match historical data require **higher risk aversion in recessions** (i.e. state-dependent and countercyclical risk aversion) – Melino and Yang (2002), Gordon and St-Amour (2000).
- ▶ Examples:
 - Habit formation – Campbell and Cochrane (1999).
 - Time varying loss aversion – Barberis, Huang and Santos (2001).

2 Major Contributions

- ▶ Defining a new set of preferences, exhibiting state-dependent, counter-cyclical risk aversion.
- ▶ Showing how this can resolve the equity premium puzzle.

3 Generalized Disappointment Aversion

- ▶ Specific example of recursive preferences – Epstein and Zin (1989).



- Aggregator

*

$$W(c, z) = \left[\left(1 - \frac{1}{1 + \rho}\right) c^\gamma + \frac{1}{1 + \rho} z^\gamma \right]^{1/\gamma}.$$

* Certainty equivalent (CE) defined by:

$$\underbrace{u(\mu_t(x_{t+1}))}_{\text{utility of CE}} = \underbrace{E_t u(x_{t+1})}_{\text{expectation of utility of gamble}} - \underbrace{\beta E_t [u(\delta\mu_t(x_{t+1})) - u(x_{t+1}) I(x_{t+1} \leq \delta\mu_t(x_{t+1}))]}_{\text{penalty function}},$$

where $u(x) = x^\alpha / \alpha$.

- For $\beta > 0$, if the outcome x_{t+1} is 'disappointing', i.e. $x_{t+1} \leq \delta\mu_t(x_{t+1})$, the utility of the CE is reduced by the penalty function.
- β is the weight attached to this penalty function – more positive β means a higher penalty.

- δ fixes how easily disappointed the agent is relative to the certainty equivalent. Higher δ means agents are more easily disappointed.
- Relative risk aversion $1 - \alpha$, Elasticity of intertemporal substitution $1 / (1 - \gamma)$.

3.1 Advantages of this approach

- The disappointment benchmark is determined endogenously — different from loss aversion.
- Introducing new preference parameters – link to standard time-separable preferences and more general recursive preferences made clear.
- Any reverse engineering is far from obvious.

4 Comments and questions

- ▶ No closed form solutions – the CE is the solution to fixed point problem.
 - Therefore difficult to see exactly how GDA affects the pricing kernel
 - * For zero relative risk aversion ($\alpha = 1$) and perfectly elastic intertemporal substitution ($\gamma = 1$):

$$M_{t+1} = \frac{1}{1 + \rho} \frac{1 + \beta I \left(\frac{R_{t+1}^x}{1 + \rho} < \delta \right)}{1 + \beta \delta P \left(\frac{R_{t+1}^x}{1 + \rho} < \delta \mid \mathcal{I}_t \right)},$$

where R_{t+1}^x is the return on the claim to the consumption endowment (to be determined *endogenously* in equilibrium).

- ▶ Can we obtain closed-form solutions in continuous-time for specific parameter values?
 - Thus obtain expressions for CAPM, stock returns, riskless rate, stock return volatility.

4.1 Outline of possible approach

- ▶ Stochastic differential utility with aggregator (W, μ) , where

$$W(c, z) = \left[\left(1 - \frac{1}{1 + \rho} \right) c^\gamma + \frac{1}{1 + \rho} z^\gamma \right]^{1/\gamma}$$

and μ is defined by the fixed point problem:

$$\int H(x, \mu(p)) dp(x) = 0,$$

where

$$H(x, y) = u(x) - \beta \{u(\delta y) - u(x)\} I(x \leq \delta y) - u(y).$$

Note that H is discontinuous at $x = \delta y$.

- ▶ Characterize utility process U_t as solution to:

$$U_t = E_t \int_t^T g(U_s) dL_s + W(c_s, V_s) ds,$$

where L_t is local time process of $(U_t, \mu(\sim U_t))$, g to be determined by applying Ito's Lemma to $H(U_t, \mu(\sim U_t))$.

- ▶ Discontinuity in H (first order risk aversion) \rightarrow *singular control problem*.
- See if a closed form solution can be obtained for special cases, such as

$$\alpha = 1, \rho = 1, \beta > 0, \delta = 1.$$

or

$$\alpha = 1, \rho = 1, \beta > 0, \delta = 0.$$

- Use asymptotic analysis to obtain local extensions of solutions for special cases.