Discussion: Generalized Disappointment Aversion and Asset Prices

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1 Motivation

- Equity premium puzzle – Mehra and Prescott (1985)

1.1 State-dependent and countercyclical risk aversion

- For a representative agent economy to match historical data require **higher risk aversion in recessions** (i.e. state-dependent and countercyclical risk aversion) – Melino and Yang (2002), Gordon and St-Amour (2000).

- Examples:
  - Habit formation – Campbell and Cochrane (1999).
  - Time varying loss aversion – Barberis, Huang and Santos (2001).

2 Major Contributions

- Defining a new set of preferences, exhibiting state-dependent, counter-cyclical risk aversion.

- Showing how this can resolve the equity premium puzzle.

3 Generalized Disappointment Aversion

- Specific example of recursive preferences – Epstein and Zin (1989).

$$\left.Recursiveclass\right\} \left.Dekel-Chewclass\right\} \left.GDA\right\} \left.DA\right\} \left.\begin{array}{l} Kreps-Porteusutility \\ Standardadditiveutility \end{array}\right\}$$

- Aggregator
  - $$W(c,z) = \left[\left(1 - \frac{1}{1+\rho}\right)c^{\gamma} + \frac{1}{1+\rho}z^{\gamma}\right]^{1/\gamma}.$$
  - Certainty equivalent (CE) defined by:

$$\underbrace{u(\mu_t(x_{t+1}))}_{utility of CE} = \underbrace{E_t u(x_{t+1})}_{expectation of utility of gamble} - \underbrace{\beta E_t\left[u(\delta\mu_t(x_{t+1})) - u(x_{t+1})\left[I(x_{t+1} \leq \delta\mu_t(x_{t+1}))\right]\right]}_{penalty function},$$

  where $u(x) = x^{\alpha}/\alpha$.
- For $\beta > 0$, if the outcome x_{t+1} is 'disappointing', i.e. $x_{t+1} \leq \delta\mu_t(x_{t+1})$, the utility of the CE is reduced by the penalty function.
- β is the weight attached to this penalty function – more positive β means a higher penalty.

- δ fixes how easily disappointed the agent is relative to the certainty equivalent. Higher δ means agents are more easily disappointed.
- Relative risk aversion $1-\alpha$, Elasticity of intertemporal substitution $1/\left(1-\gamma\right).$

3.1 Advantages of this approach

- The disappointment benchmark is determined endogenously — different from loss aversion.
- Introducing new preference parameters – link to standard time-separable preferences and more general recursive preferences made clear.
- Any reverse engineering is far from obvious.

4 Comments and questions

- No closed form solutions – the CE is the solution to fixed point problem.
  - Therefore difficult to see exactly how GDA affects the pricing kernel
    - For zero relative risk aversion ($\alpha = 1$) and perfectly elastic intertemporal substitution ($\gamma = 1$):

$$M_{t+1} = \frac{1}{1+\rho} \frac{1 + \beta I\left(\frac{R^x_{t+1}}{1+\rho} < \delta\right)}{1 + \beta\delta P\left(\frac{R^x_{t+1}}{1+\rho} < \delta \,\middle|\, \mathcal{I}_t\right)},$$

where R^x_{t+1} is the return on the claim to the consumption endowment (to be determined *endogenously* in equilibrium).

- Can we obtain closed-form solutions in continuous-time for specific parameter values?
  - Thus obtain expressions for CAPM, stock returns, riskless rate, stock return volatility.

4.1 Outline of possible approach

- Stochastic differential utility with aggregator (W, μ),where

$$W(c, z) = \left[\left(1 - \frac{1}{1+\rho}\right) c^{\gamma} + \frac{1}{1+\rho} z^{\gamma}\right]^{1/\gamma}$$

and μ is defined by the fixed point problem:

$$\int H(x, \mu(p))\, dp(x) = 0,$$

where

$$H(x, y) = u(x) - \beta \{u(\delta y) - u(x)\} I(x \leq \delta y) - u(y).$$

Note that H is discontinuous at $x = \delta y$.

- Characterize utility process U_t as solution to:

$$U_t = E_t \int_t^T g\left(U_s\right) dL_s + W\left(c_s, V_s\right) ds,$$

where L_t is local time process of $\left(U_t, \mu\left(\sim U_t\right)\right)$, g to be determined by applying Ito's Lemma to $H\left(U_t, \mu\left(\sim U_t\right)\right)$.

- Discontinuity in H (first order risk aversion)$\rightarrow$ *singular control problem*.
  - See if a closed form solution can be obtained for special cases, such as

$$\alpha = 1, \rho = 1, \beta > 0, \delta = 1.$$

or

$$\alpha = 1, \rho = 1, \beta > 0, \delta = 0.$$

  - Use asymptotic analysis to obtain local extensions of solutions for special cases.